Modular Embedding of Problems onto Quantum Annealers

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Motivation

- Quantum annealers benefit from high qubit connectivity
- “Chains” of physical qubits used to represent single virtual qubits
- Embedding takes problems onto current topology is NP-Hard
- Previous work[1] shows improvement for specific problem by adding extra structural constraints
- Separates the problem into cells of 2 qubits
- Cells can only be embedded onto certain physical qubits on the Chimera structure
- Similar to the idea behind DWave’s Locally Structured Embeddings

NchooseK

NchooseK is a domain-specific, constraint-based language built for automatically setting up problems for both gate-based machines and quantum annealers[2].

- Good candidates for modular embedding, thanks to constraint-based nature
- NchooseK uses constraints which say “Of N variables, K must be true”
- Constraints take the form nchoosek([N1, N2], [K1, K2])
- Many NP problems have been solved with NchooseK
- One-hot encoding problems particularly suited to this type of embedding
- Several qubits represent one variable
  - Qubit measured as |1> indicates “hot” value
- Map coloring problem good example here
  - Map coloring uses 2 kinds of constraints, shown below:
    - Circles represent variables (regions P and Q)
    - Boxes represent constraints, number shows K for that constraint
  - One constraint per node to ensure one color per node:
    - nchoosek(a, 1, a, a, a, 3, 3)
  - n constraints per edge ensuring two nodes of the same color not connected:
    - nchoosek(a, b, c, 1)

Connectivity of central cells on section of DWave Pegasus architecture
Abstract representation in upper right corner
Thick colored line indicates chain representing single virtual qubit

NchooseK Usage

- NchooseK is used to embed problems onto quantum annealers
- Constraints take the form nchoosek([N1, N2], [K1, K2])
- Constraints can be added through implications of other constraints
- Cells can only be embedded onto certain physical qubits on the Chimera structure
- “Chains” of physical qubits used to represent single virtual qubits

Methodology

We decided to start our investigation with the Map Color and Clique Cover problems due to their one-hot encoding lending to distinct cells with predictable connections between them.

- First tried DWave’s current Pegasus architecture.
- Tried to find good encodings for maps of 3 and 4 colors
- These cells need the following properties:
  - Each cell must be a clique
    - Each qubit within the cell connected to every other qubit within the cell
    - Corresponding qubits in connected cells must be connected to each other
    - Ex: P, connected to Q in NchooseK example
  - Cell for 3 colors use 4 qubits (bottom left)
    - Thicker line indicates a chain, representing one virtual qubit
    - Cells have a degree of 4
    - Two cells found for 4 colors
    - One version uses 8 qubits per cell (bottom center)
      - Cells have a degree of 6
    - Other version uses 4 qubits per cell (bottom right, unused)
      - Corresponding qubits have degree of 2 or 3
      - Additional constraints on placement depending on connectivity
      - Cells need to be connected to connect with cells not on the same diagonal
    - Minorminer was used to map on our abstract maps and on the full Pegasus map

Results

- Map color problem based on US continental map, starting with Tennessee
- Compared metrics of the mappings: Average/Max chain length, total number of qubits, embedding time
  - Modular (blue) performed worse than standard (red), except in embed time
  - Ran some problems on physical machines, compared correctness
- Modular mappings once again underperformed when compared to standard mapping

Conclusions

- Finding good modular maps is non-trivial
  - Modular embedding is faster but worse than full map embedding in this case
  - Modular embedding is better in other situations
  - Likely better performance with cell size 2 or different necessary connections between cells
  - Unable to take advantage of many connections on Pegasus

References/Acknowledgements

https://github.com/nchoosek

Images:
- Table: Selected results for 4-color maps of US.

- 3-color, clique cover, and DWave Chimera architecture performed similarly.

- Embedding time for Modular vs Standard Embedding

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