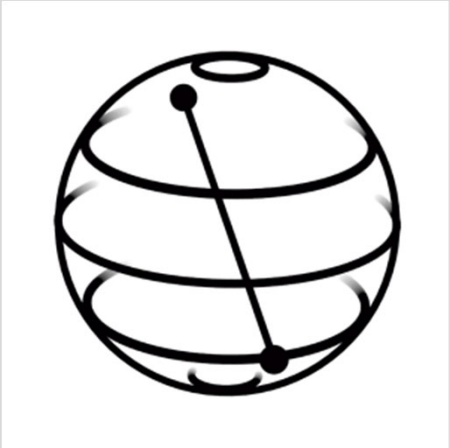

Qiskit Programming Exercise: Teleportation

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1

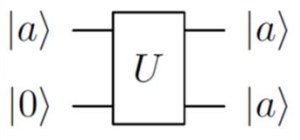
Quantum Teleportation?

- Remember "no cloning" rule?
  Can't duplicate the state of a qubit.
- Can **transmit** and **reconstruct** the state of a qubit, using two classical bits and a pair of **entangled** qubits. State of the original qubit is **destroyed**.
- This is known as "teleportation."

2

No-Cloning Principle (revisited)

Suppose we have a cloning transformation U, such that $\boldsymbol{U(|a\rangle|0\rangle) = |a\rangle|a\rangle}$ for any quantum state $|a\rangle$.



Let $|a\rangle$ and $|b\rangle$ be two orthogonal quantum states. Therefore,

$$U(|a\rangle|0\rangle) = |a\rangle|a\rangle$$
$$U(|b\rangle|0\rangle) = |b\rangle|b\rangle$$

Consider $|c\rangle = \frac{|a\rangle+|b\rangle}{\sqrt{2}}$.

By linearity:

$$U(|c\rangle|0\rangle) = \frac{1}{\sqrt{2}}\big(U(|a\rangle|0\rangle) + U(|b\rangle|0\rangle)\big)$$
$$= \frac{1}{\sqrt{2}}(|a\rangle|a\rangle + |b\rangle|b\rangle)$$

$\neq$

By definition of cloning transformation:

$$U(|c\rangle|0\rangle) = |c\rangle|c\rangle$$
$$= \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle) \otimes \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle)$$
$$= \frac{1}{2}(|a\rangle|a\rangle + |a\rangle|b\rangle + |b\rangle|a\rangle + |b\rangle|b\rangle)$$

These are not equal $\Rightarrow$ there is no U for which both can be true.

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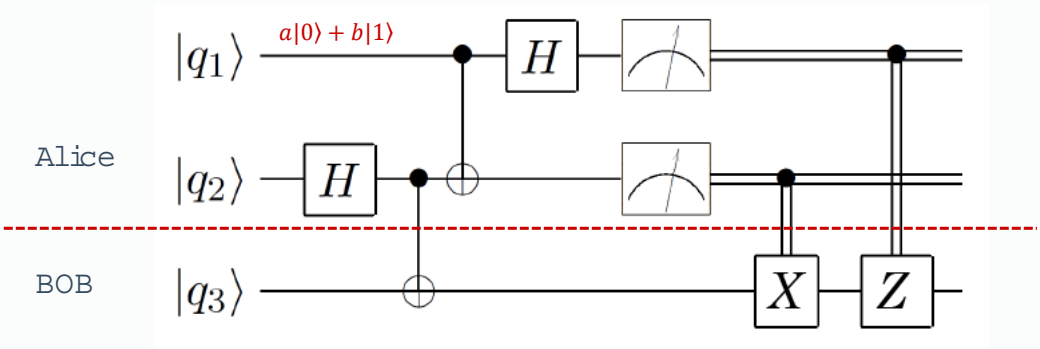
Teleportation (1)



Alice wants to send q1 to Bob. Alice does not know the state of q1.

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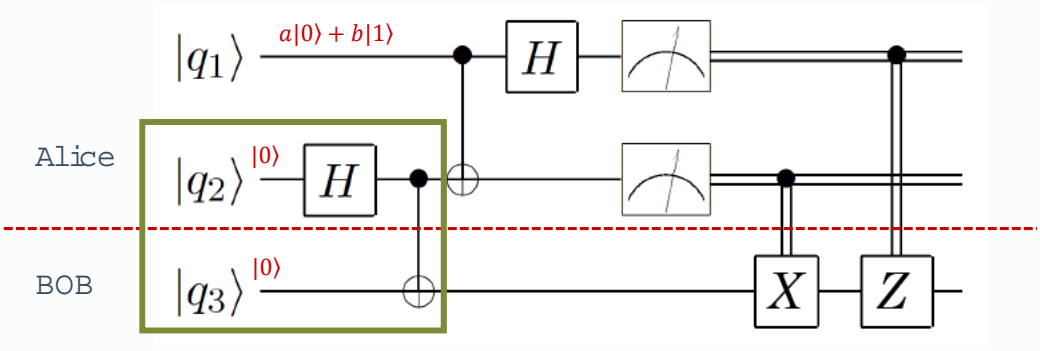
Teleportation (2)



First, an EPR pair is created – an entangled pair of qubits in state $|00\rangle + |11\rangle$.
(Leaving off normalizing coefficients for simplicity.)
Full state of 3-qubit system is $\boldsymbol{a}|\mathbf{000}\rangle + \boldsymbol{a}|\mathbf{011}\rangle + \boldsymbol{b}|\mathbf{100}\rangle + \boldsymbol{b}|\mathbf{111}\rangle$

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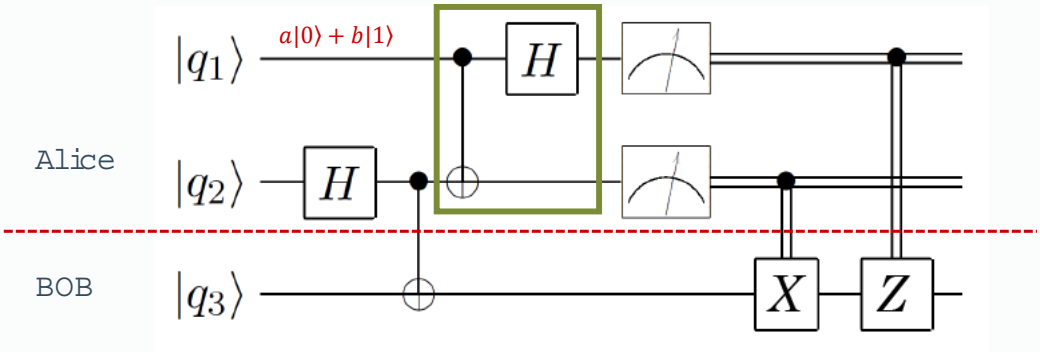
Teleportation (3)



Next, Alice performs operations on her two qubits, and state becomes:

$$\boldsymbol{a}(|\mathbf{000}\rangle + |\mathbf{011}\rangle + |\mathbf{100}\rangle + |\mathbf{111}\rangle) + \boldsymbol{b}(|\mathbf{010}\rangle + |\mathbf{001}\rangle - |\mathbf{110}\rangle - |\mathbf{101}\rangle)$$

$$|\mathbf{00}\rangle(\boldsymbol{a}|\mathbf{0}\rangle + \boldsymbol{b}|\mathbf{1}\rangle) + |\mathbf{01}\rangle(\boldsymbol{a}|\mathbf{1}\rangle + \boldsymbol{b}|\mathbf{0}\rangle) + |\mathbf{10}\rangle(\boldsymbol{a}|\mathbf{0}\rangle - \boldsymbol{b}|\mathbf{1}\rangle) + |\mathbf{11}\rangle(\boldsymbol{a}|\mathbf{1}\rangle - \boldsymbol{b}|\mathbf{0}\rangle)$$

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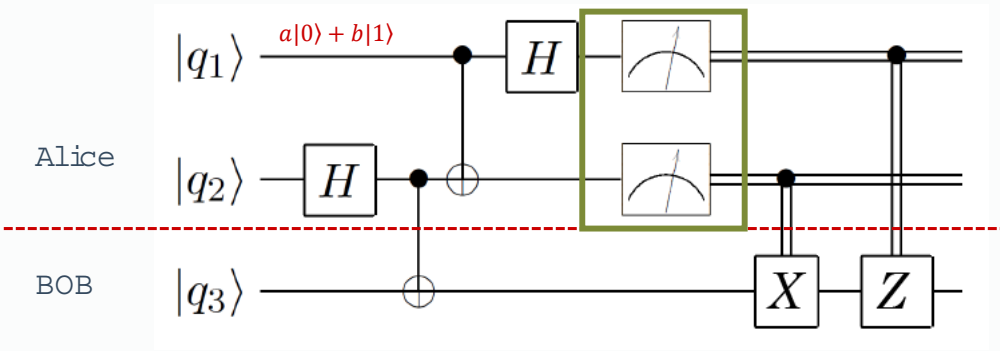
Teleportation (4)



Alice measures her two bits, which forces Bob's bit into one of the four states.

$$|00\rangle(a|0\rangle + b|1\rangle) + |01\rangle(a|1\rangle + b|0\rangle) + |10\rangle(a|0\rangle - b|1\rangle) + |11\rangle(a|1\rangle - b|0\rangle)$$

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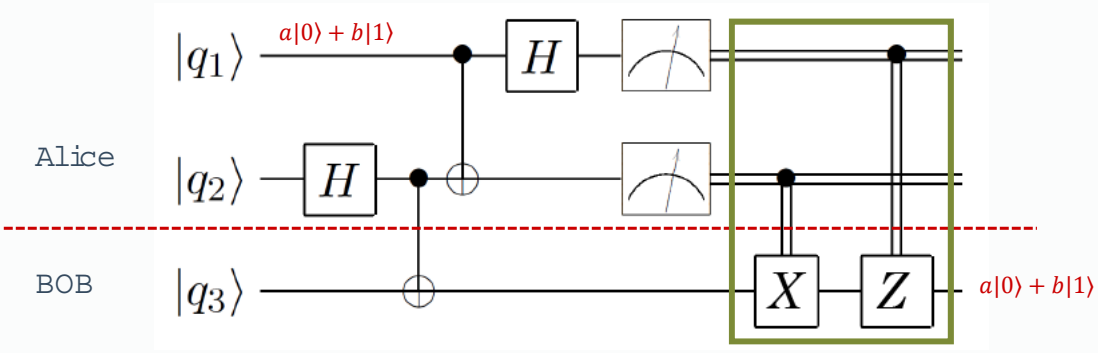
Teleportation (5)



The two measure bits (classical) are sent to Bob. At that point, he knows the state of his qubit, and he "corrects" it using X, Z, or both (Y), to recover the original state.

$$|00\rangle(a|0\rangle + b|1\rangle) + |01\rangle(a|1\rangle + b|0\rangle) + |10\rangle(a|0\rangle - b|1\rangle) + |11\rangle(a|1\rangle - b|0\rangle)$$

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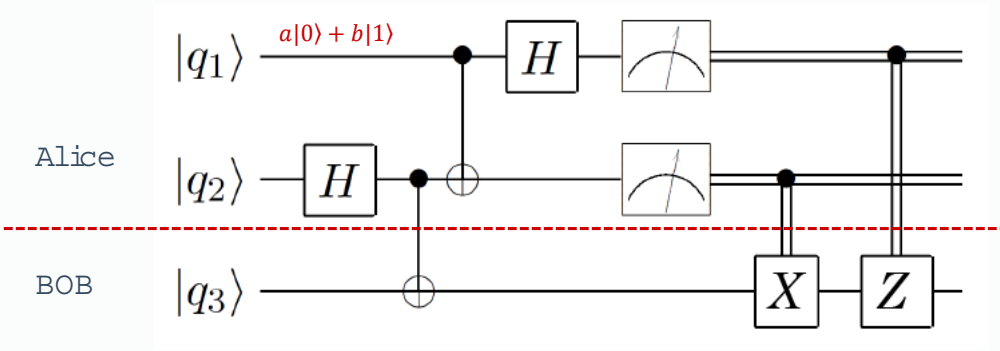
Assignment



Use qiskit to implement the circuit and demonstrate that it works.
(Only works with simulator)

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