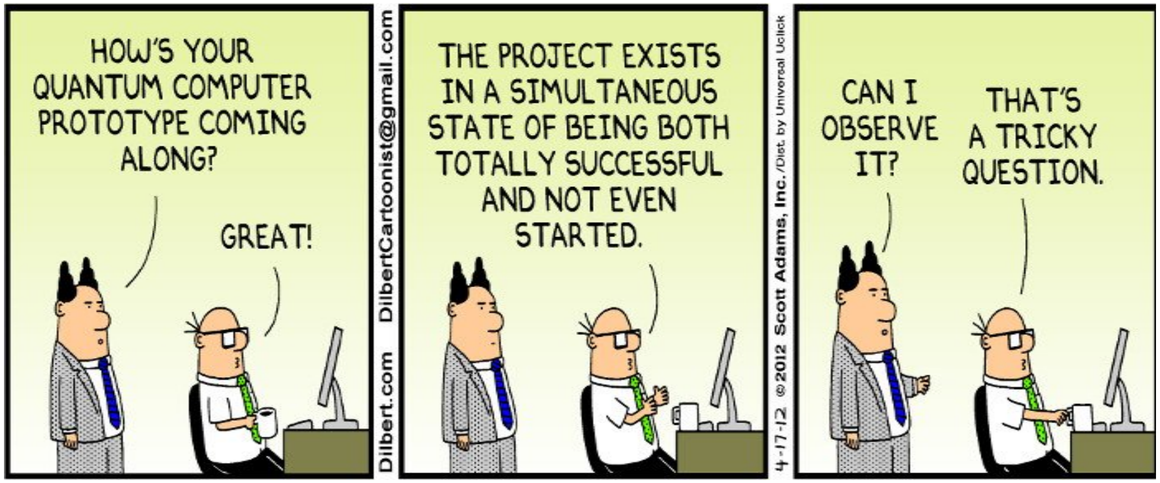
Building Blocks for Quantum Computing Part V Operation of the Trapped Ion Quantum Computer



Patrick Dreher

CSC801 – Seminar on Quantum Computing

Spring 2018

Goal Is To Understand The Principles And Operation of the Trapped Ion Quantum Computer (TIQC)

- Building Blocks for Quantum Computing Part I
- Building Blocks for Quantum Computing Part II
- Building Blocks for Quantum Computing Part III
  Quantum Mechanics Primer
- Building Blocks for Quantum Computing
  Design and Construction of the TIQC – Part IV
- Building Blocks for Quantum Computing
  Operation of the TIQC – Part V

Demonstrate That a TIQC Based on The Ideas of Quantum Computing Can Be Constructed and Operated in the Lab

Criteria That Must be Satisfied in Order to Operate a Trapped Ion Quantum Computer

1. System must contain a 2 qubit quantum system
2. System must be able to be initialized to a single initial qubit state before quantum computations begin
    a. Cooling the experimental apparatus
    b. Optical pumping to initialize the TIQC
3. System must be able to perform a coherent set of universal quantum gate operations (single and 2 qubit)
4. Qubits in the system must be measurable
5. System must be scalable

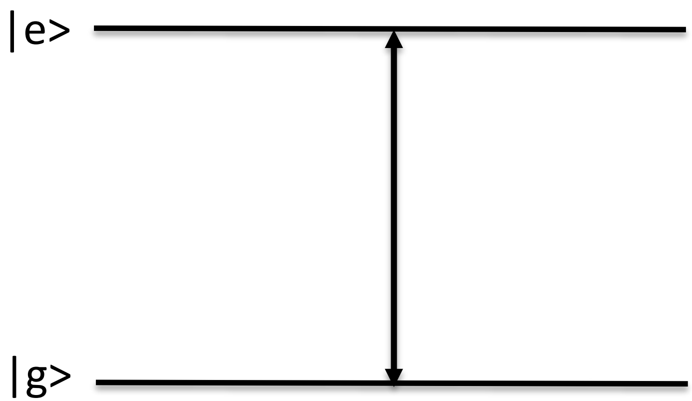
1. System Must Be Able to Identify a Two Qubit System

- Candidate material is a single 2 level spin system interacting with an electromagnetic field
  a) Qubit can be identified by the quantized bound states of an atomic material as seen through ability of a spin to respond to an electromagnetic field
  b) A second qubit can be identified through the set of interactions of the ion's vibrational modes
     i. Spin is physically confined within a harmonic potential such that it becomes quantized with energy of scale hν
     ii. These qubits are identified as center of mass phonon vibrations

Outer Product Representation of Available Qubit Quantum States in a TIQC

Two level Ion

Harmonic Potential Trap



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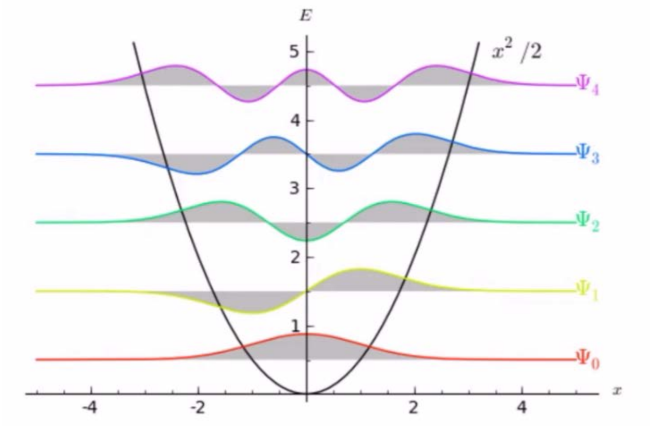

2. System Must be Able to be Initialized to a Single Qubit State before Quantum Computations Begin

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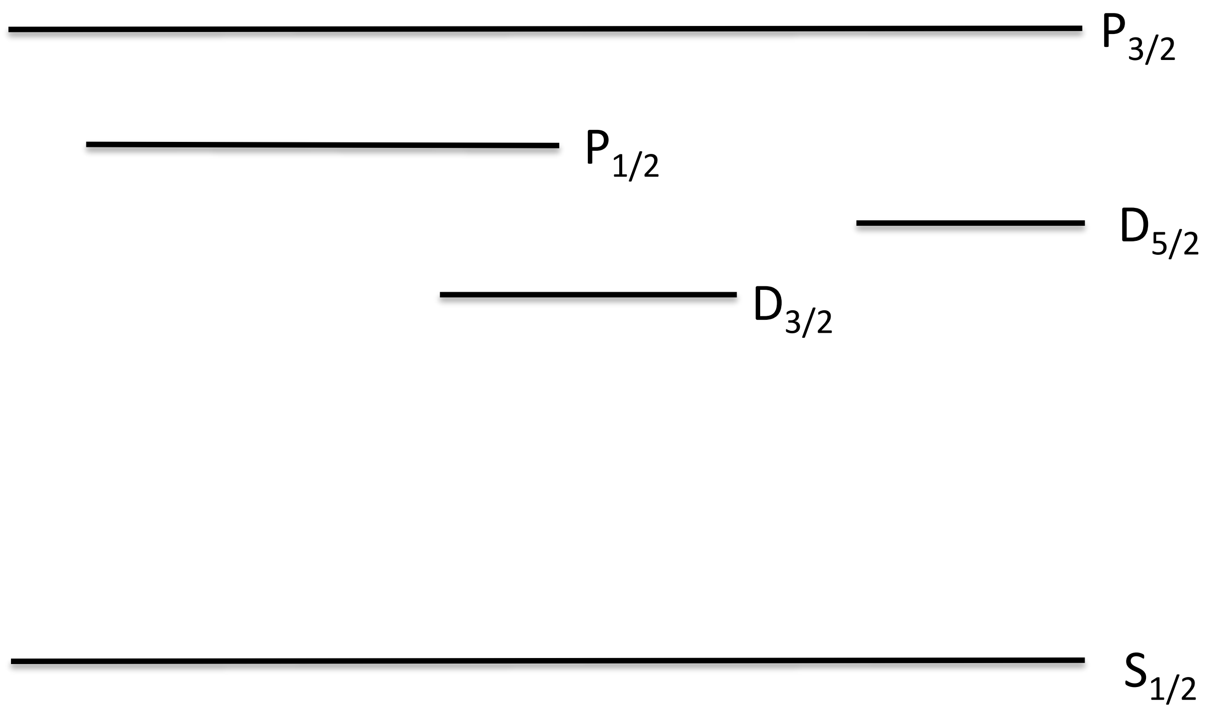
- Experimental apparatus enclosing the system must be cooled to near absolute zero so that the Hilbert space of accessible quantum states can be reduced to a 2 state system for each qubit (ground |g> and excited |e> state)

2. System Must be Able to be Initialized to a Single Qubit State before Quantum Computations Begin

- Identify relevant energy states of $^{40}Ca^{+}$



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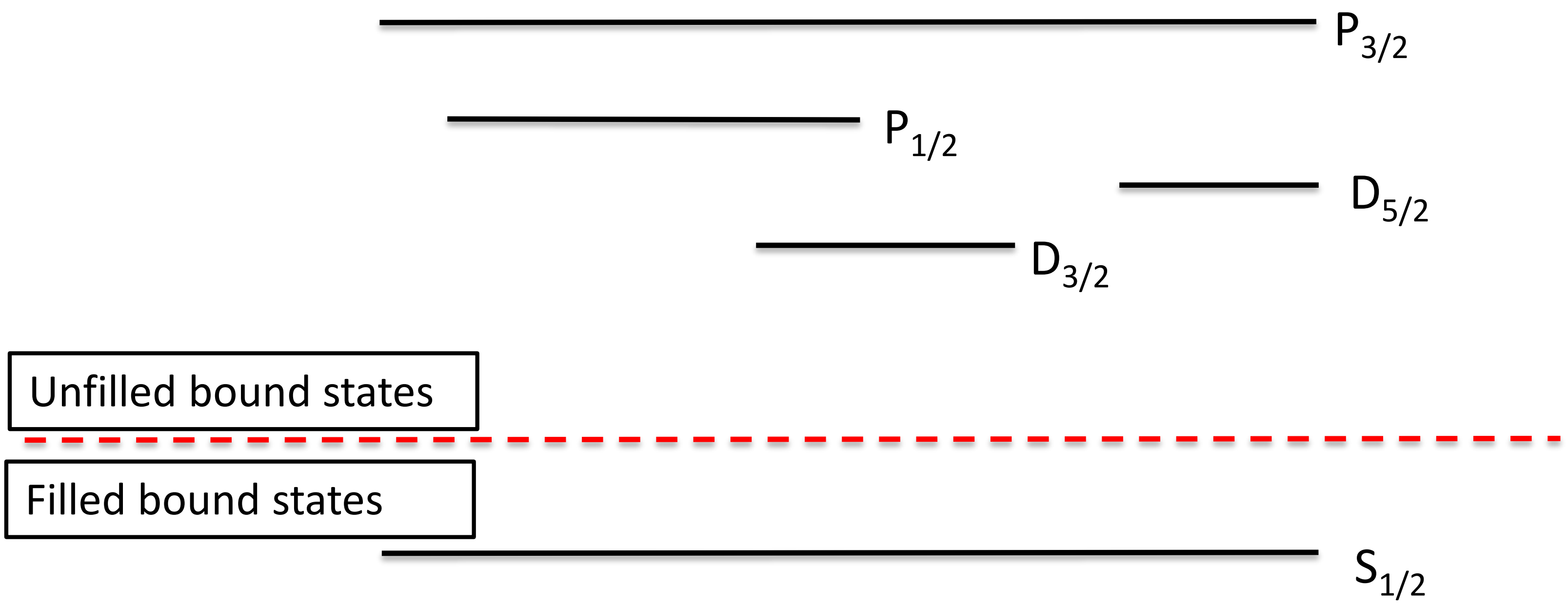
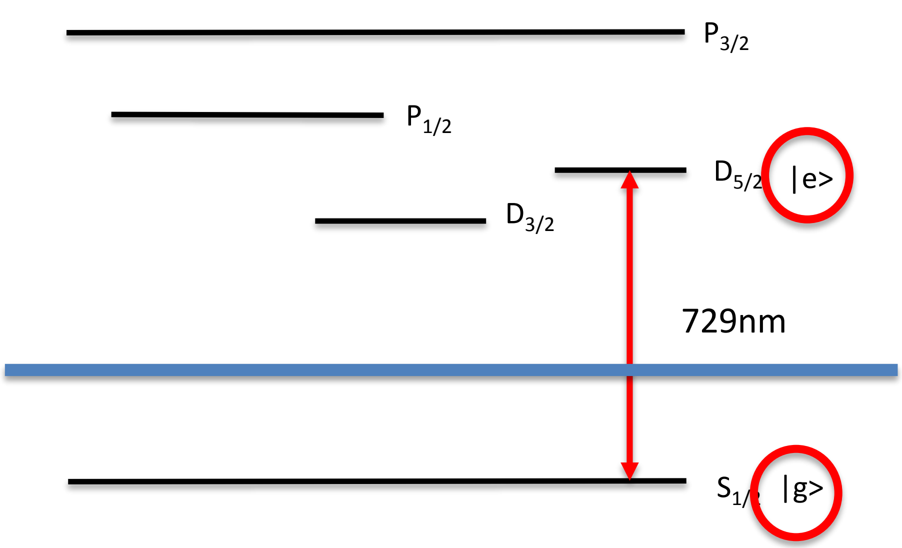


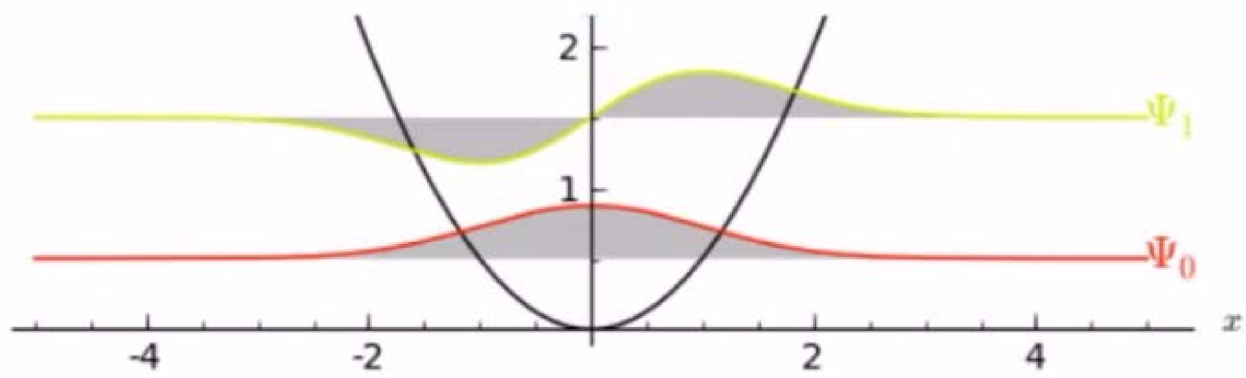
Building a Long Lived Ion Qubit State

- Now use another laser tuned to the resonant wavelength of (729 nm) to force an excited state population into the $D_{5/2}$ state |e> from the $S_{1/2}$ ground state |g>
- From laws of QM this is a forbidden transition and so the excited state will be long lived (~1 sec) compared to the lifetime of an allowed transition (~ 1 nanosecond) → stable qubit
- Now have constructed a long-lived stable qubit



Construct the Lowest Vibrational States in the Harmonic Oscillator Trap

- Associated with the spin of the electron there is also a vector coupling to the nuclear spin (hyperfine interaction)
- Associated with vibrational states
- The vibrational states of the ion can split into a 2 level system associated with the hyperfine interaction if the material sample is cooled to near absolute zero



2a. - Initialization Requirement for a Low Temperature Experimental Environment

Requirement for a Low Temperature Experimental Apparatus

- At room temperature the electrons are subject to many types of energy fluctuations
- Above the filled electron shells, there are many unfilled bound states to which the electron can transition (unwanted volunteers)
- Must suppress this “jitter” so that the number and type of transitions between bound states in the Ca atom is minimized and controlled
- Cool the apparatus to limit the size of the Hilbert space available to the qubit

2a. - Initialization Requirement for a Low Temperature Experimental Environment

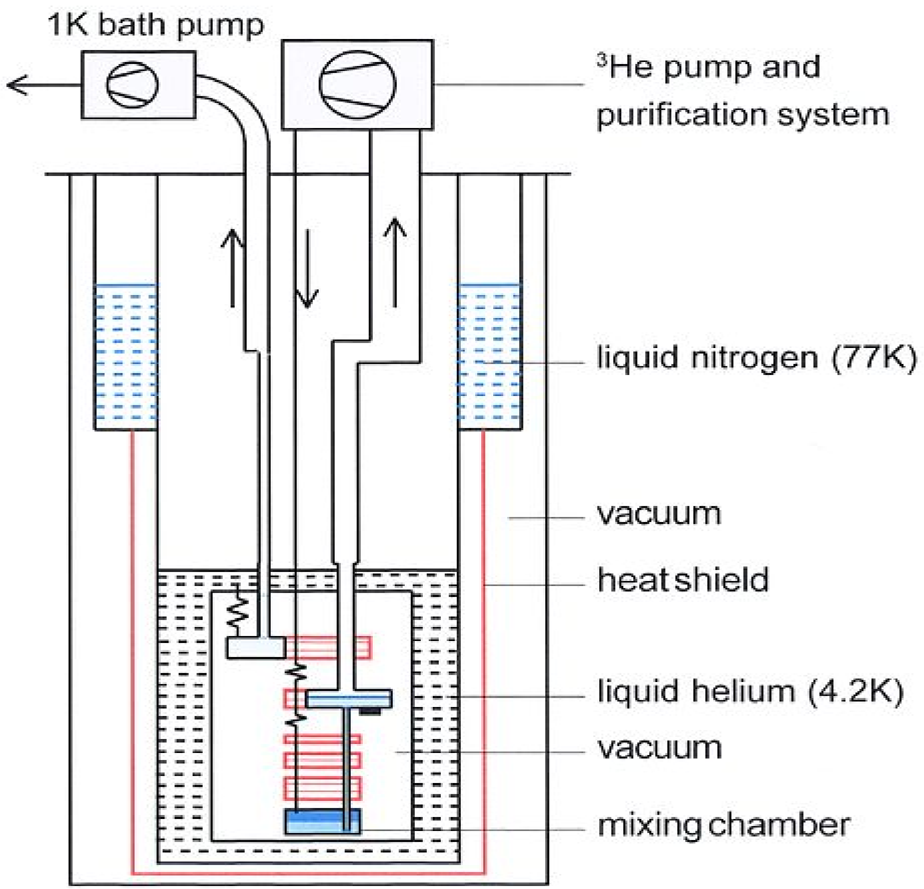
- Required to isolate and cool the ions in the experimental apparatus such that
  1. Only a few excited states above the ground state are accessible to the ion (limit the size of the Hilbert space available for energy transitions)
  2. The trap forms a 1D harmonic oscillator potential with the (phonon) vibrational states sufficiently cooled so that only the lowest states are accessible
- This will set the ions in an initial state for the computation

2a - Initialization Requirement for a Low Temperature Experimental Environment *

- To generate that initial state of quantum behavior the ion trap environment is cooled with
  - Dilution Refrigerators
  - Doppler Cooling
  - Sideband Cooling

* See lecture slides DREHER – Building Blocks of Quantum Computing - Part IV

Dilution Refrigerator *



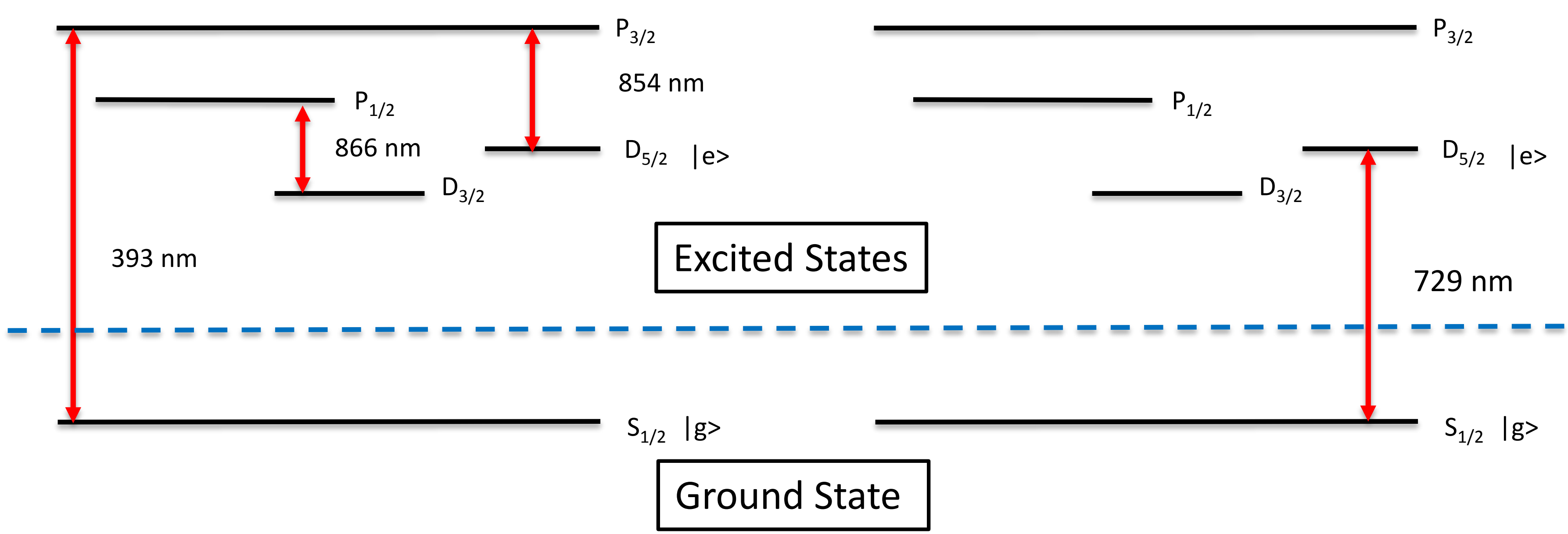
* Image from http://www.wikiwand.com/en/Dilution_refrigerator

IBM Q Quantum Computer Cryostat

2b - Initialization of the TIQC Through Optical Pumping

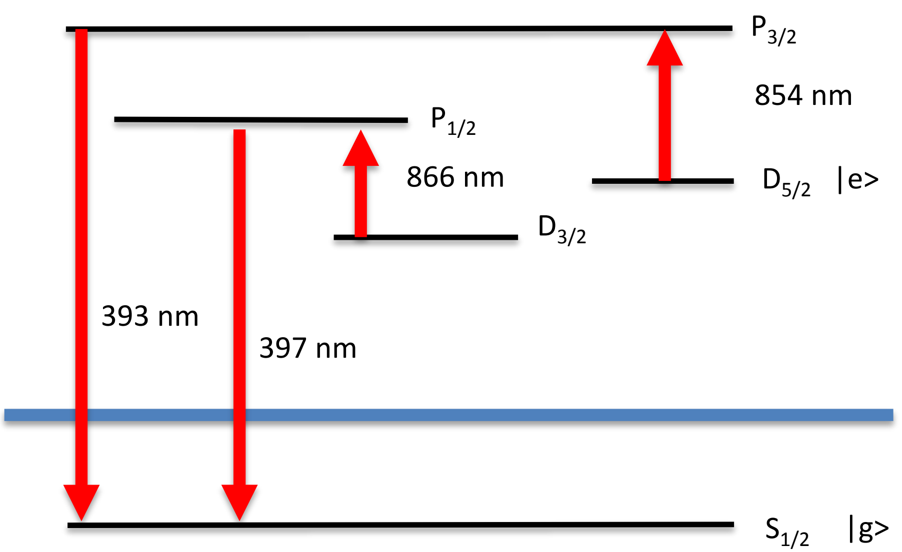
- After cooling the apparatus to limit the number of transition energy states accessible to the ion, must also clear the excited (spin) states to initialize the system
- Accomplished through pumping the states with laser light tuned to the transition wavelength (energy difference) between the excited and ground state of the ion

Identify the Lowest ^{40}Ca Energy Level Transitions Between Filled/Unfilled Bound States



Remove Unwanted Populated Excited States so that the Ion to Initially in the $S_{1/2}$ Ground State

- Optical pumping
- Depopulate the $P_{1/2} \leftrightarrow D_{3/2}$ transition that can contaminate the $D \leftrightarrow S$ long lived state for a qubit
- Also need to de-populate the $D_{5/2}$ state
- Need 2 new lasers (854 nm and 866 nm) to pump electrons to the $P_{1/2}$ and $P_{3/2}$ states that can then drain to the $S_{1/2}$ ground state



3. System Must Be Able To Perform a Coherent Set of Universal Quantum Gate Operations (Single and 2 Qubit)

TIQC Operational Procedure

TIQC Operational Procedure

- A quantum computer can be programmed to simulate general computational problems using a combination of single qubit and universal CNOT operations
- Outline the experimental procedure for operating a TIQC
- Goal is to be able to experimentally generate 1 qubit gates and 2 qubit CNOT gates

Recall Discussion that Single Qubit Gates Constructed from Laser Pulses

Pauli X –[X]– $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_x$

Pauli Y –[Y]– $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \sigma_y$

Pauli Z –[Z]– $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma_z$

Pauli Spin Matrices*

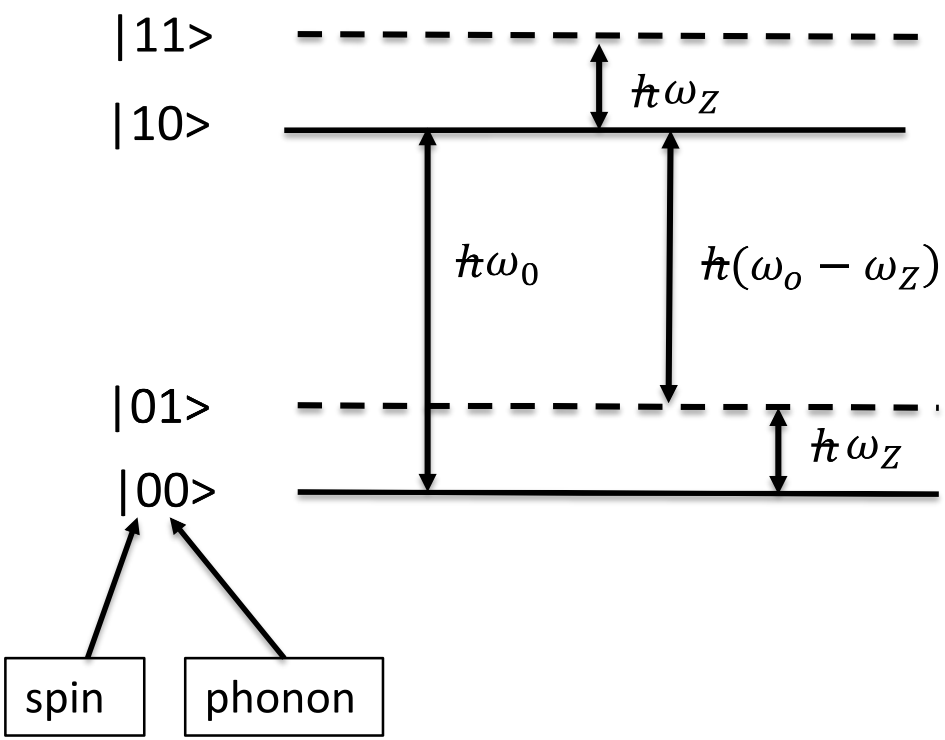
Phase –[S]– $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

$\frac{\pi}{8}$ –[T]– $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$

Hadamard –[H]– $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

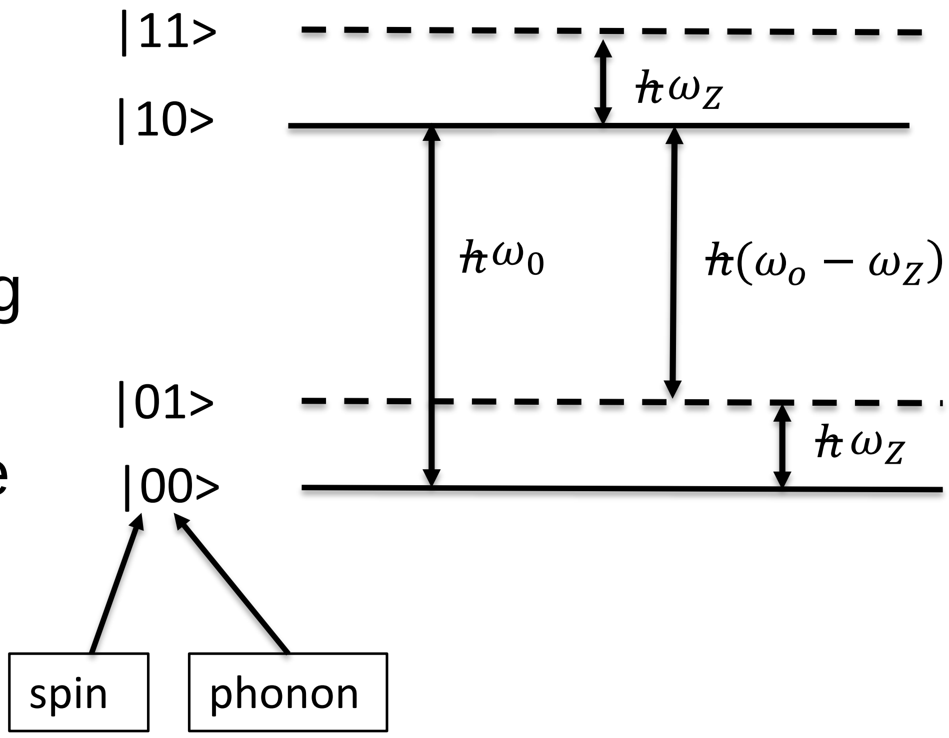
Energy Levels of a Single Atom

- Assume ion is cooled so that it is near its lowest vibrational state
- Have a ground and excited spin state and a ground and excited vibrational phonon state



Energy Levels of a Single Atom

- Use a laser detuned from the $\hbar\omega_0$ spin transition tuned to |10>→|01> transition energy $\hbar(\omega_o - \omega_Z)$
- Uniquely forces a transition from |10> → |01> without possibility of inducing other transitions
- This places the entire ion chain in the first excited vibrational state of spin |0>



For a CNOT Gate Need 2 Qubits

- Construct a set of basis vectors from a linear vector space describing wavefunction of two ions (A and B) and a collective phonon vibrational state

$$|0_A > |0_B > |0 >$$

$$|0_A > |1_B > |0 >$$

$$|1_A > |0_B > |0 >$$

$$|1_A > |1_B > |0 >$$

Laser Pulse Generates a π Pulse Directed to Ion A

- Construct operator U_A that generates a π pulse directed to ion A with energy $\hbar(\omega_o - \omega_Z)$
- Generates Rabi oscillations

$$|0_A>|0_B>|0> \longrightarrow |0_A>|0_B>|0>$$

$$|0_A>|1_B>|0> \longrightarrow |0_A>|1_B>|0>$$

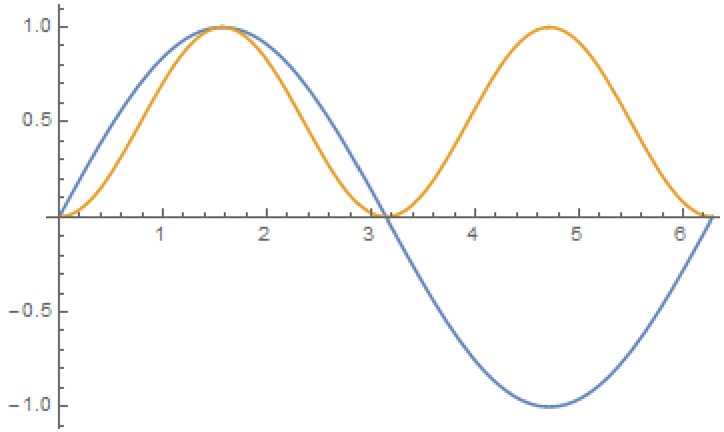
$$|1_A>|0_B>|0> \longrightarrow -i\ |0_A>|0_B>|1>$$

$$|1_A>|1_B>|0> \longrightarrow -i\ |0_A>|1_B>|1>$$

- Ion A generates phase *–i, changes ion A from |1>→|0> and phonon vibrational state |0>→|1> (ion B unaffected)*

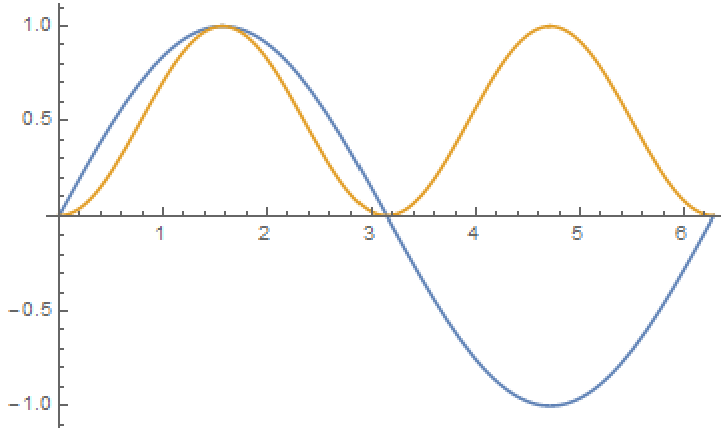
Rabi Oscillations

- Rabi oscillations (also known as the Rabi cycle or Rabi flop) is the cyclic behavior of a two-level quantum system in the presence of an oscillatory driving field such as a laser pulse
- Figure below shows cyclic probability amplitude (blue) and the measurement probability (yellow)



Rabi Oscillations Information Used to Create a Phase Gate

- Rabi oscillation after one period has changed the phase of the probability amplitude by π (phase is -*i*) (blue)
- Quantities measured in the lab are the probabilities (yellow)
- Figure shows that after the system has returned to the original state the probability has shifted by 2π but the phase by π (-*i*)
- 2π pulse in population shifts phase of wavefunction by π (-*i*)



Generate Laser Pulse Directed to Ion B

- Construct operator V_B that generates a π pulse directed to ion B and changes the phase of the wavefunction by π

$$|0_A > |0_B > |0 > \longrightarrow |0_A > |0_B > |0 >$$

$$|0_A > |1_B > |0 > \longrightarrow |0_A > |1_B > |0 >$$

$$-i\ |0_A > |0_B > |1 > \longrightarrow i\ |0_A > |0_B > |1 >$$

$$-i\ |0_A > |1_B > |1 > \longrightarrow -i\ |0_A > |1_B > |1 >$$

- Occurs only if ion B is the ground state |0> and the phonons are in excited vibrational state |1>

Apply Operator U_A a Second Time with a π Pulse Directed to Ion A

- π pulse again directed to ion A

$$|0_A>|0_B>|0> \longrightarrow |0_A>|0_B>|0>$$

$$|0_A>|1_B>|0> \longrightarrow |0_A>|1_B>|0>$$

$$i\ |0_A>|0_B>|1> \longrightarrow |1_A>|0_B>|0>$$

$$-i\ |0_A>|1_B>|1> \longrightarrow -|1>|1_B>|0>$$

- If ion A is in state |0> generates a phase rotation of –*i* and changes the state of ion A from |0>→|1> and the vibrational phonon state from |1>→|0>

Program a Sequence of Operations that will Construct a CNOT Gate

- Take the sequence of these U_A and V_B transitions and form a product $W = U_A V_B U_A$
- Recall the initial state

$$|0_A > |0_B > |0 >$$
$$|0_A > |1_B > |0 >$$
$$|1_A > |0_B > |0 >$$
$$|1_A > |1_B > |0 >$$

- Compare to the final state

$$|0_A > |0_B > |0 >$$
$$|0_A > |1_B > |0 >$$
$$|1_A > |0_B > |0 >$$
$$-|1 > |1_B > |0 >$$

Construct a 2 Qubit Truth Table for the Product Operation $W=U_AV_BU_A$

Initial State

$|0_A>|0_B>|0>$

$|0_A>|1_B>|0>$

$|1_A>|0_B>|0>$

$|1_A>|1_B>|0>$

Final State

$|0_A>|0_B>|0>$

$|0_A>|1_B>|0>$

$|1_A>|0_B>|0>$

$-|1>|1_B>|0>$

g0 g1 e0 e1

$$W_{CPHASE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{matrix} g0 \\ g1 \\ e0 \\ e1 \end{matrix}$$

The Control Phase Gate

- In a $\binom{1}{0}, \binom{0}{1}$ basis, the Control Phase gate changes the sign of the 2nd qubit when the 1st qubit is 1

$$W_{CPHASE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

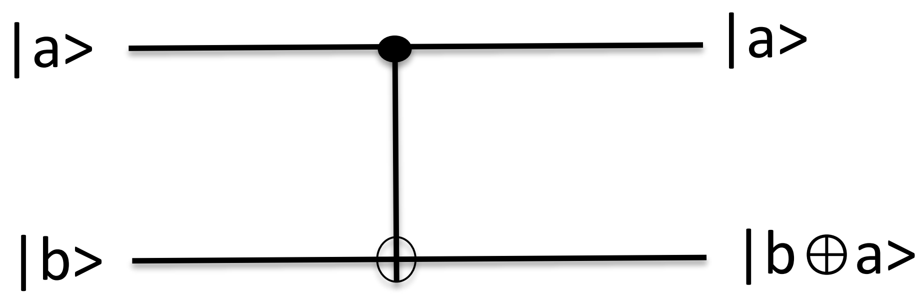
$$W_{CPHASE}^{\dagger} W_{CPHASE} = I$$

- The CPHASE gate becomes a CNOT universal quantum gate when combined with 2 Hadamard gates

Recall the Discussion of the Controlled-NOT Gate

Matrix representation of the CNOT gate

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad U^{\dagger}_{CNOT} U_{CNOT} = I$$

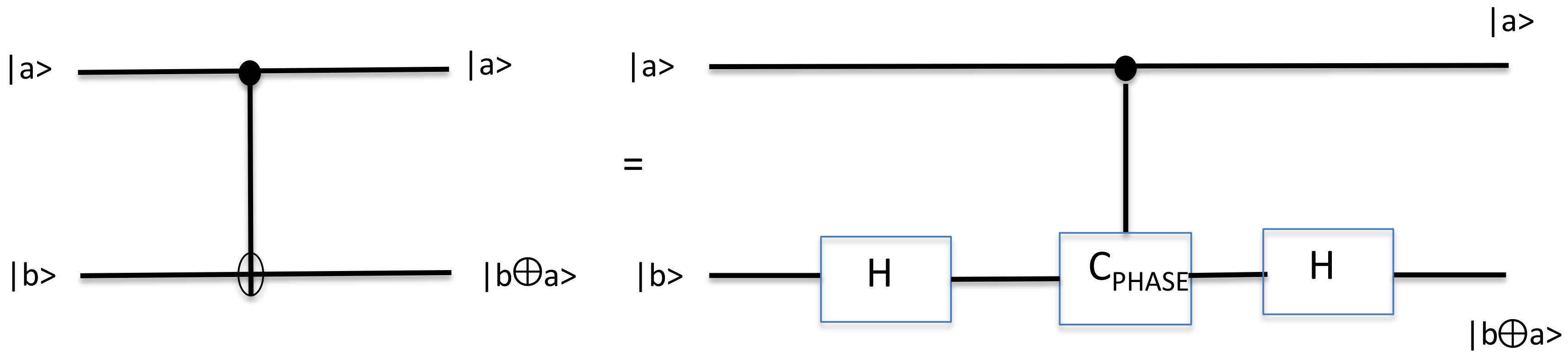


$|a\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; |b\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

|aa> → |aa>
|ab> → |ab>

|ba> → |bb>
|bb> → |ba>

Express CNOT in Terms of CPHASE



4. Qubits in The System Must Be Measurable

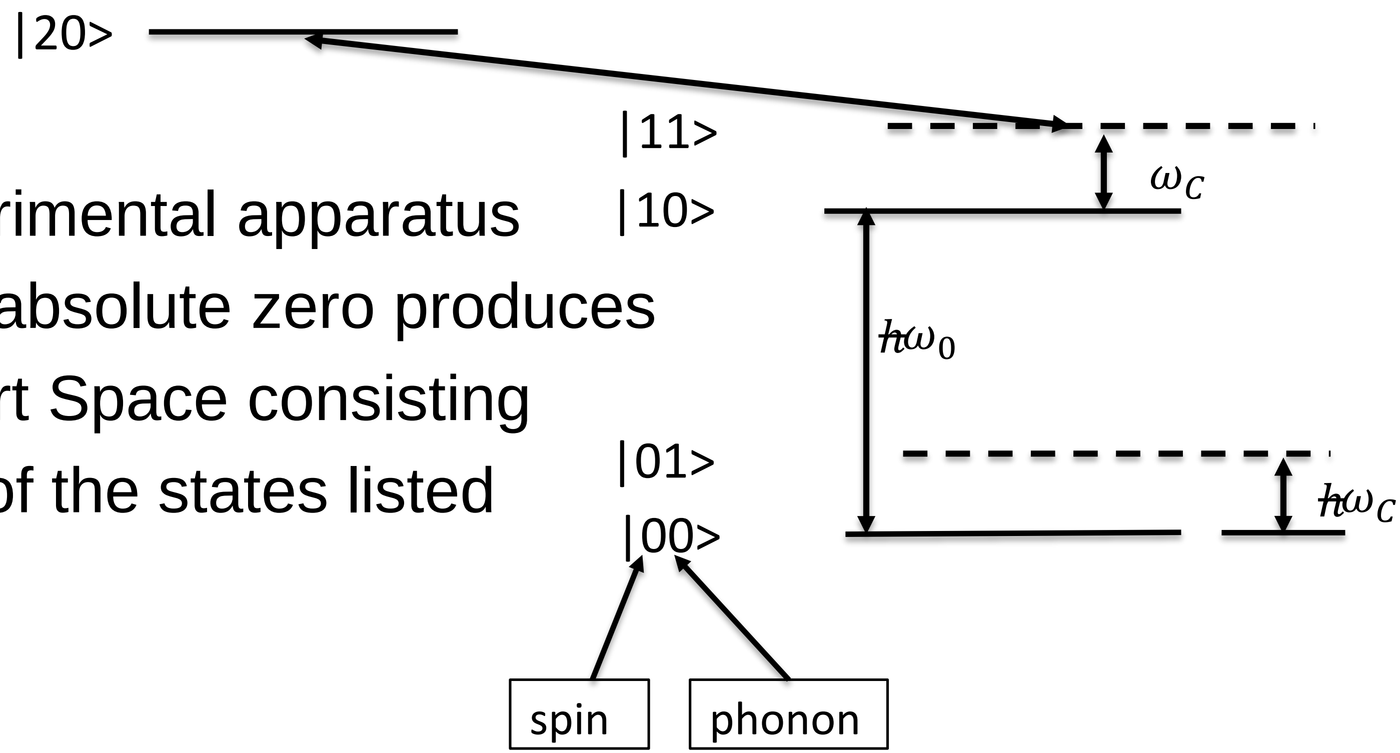
Operation of the Trapped Ion Quantum Computer

- Consider ion “A” and ion “B”
- How can spin up state of an ion be detected without collapsing the wavefunction through a measurement
- Key component is the center of mass motion of the entire ion chain in the trap
- Use laser pulses to force ion “B” into an up spin state
- If the ion B is spin up use another laser pulse to induce center of mass motion of the ion chain (common dipole motion)
- Swap the information from the up state of the ion to the center of mass motion of the ion chain – essentially communicate signal on the “data bus” of ion chain that the ion “B” is spin up
- Ion “A” detects the center of mass motion and knows that the center of mass motion is occurring because the ion “B” is in a spin up state
- Change rotation of the wavefunction but only if both ions are spin up

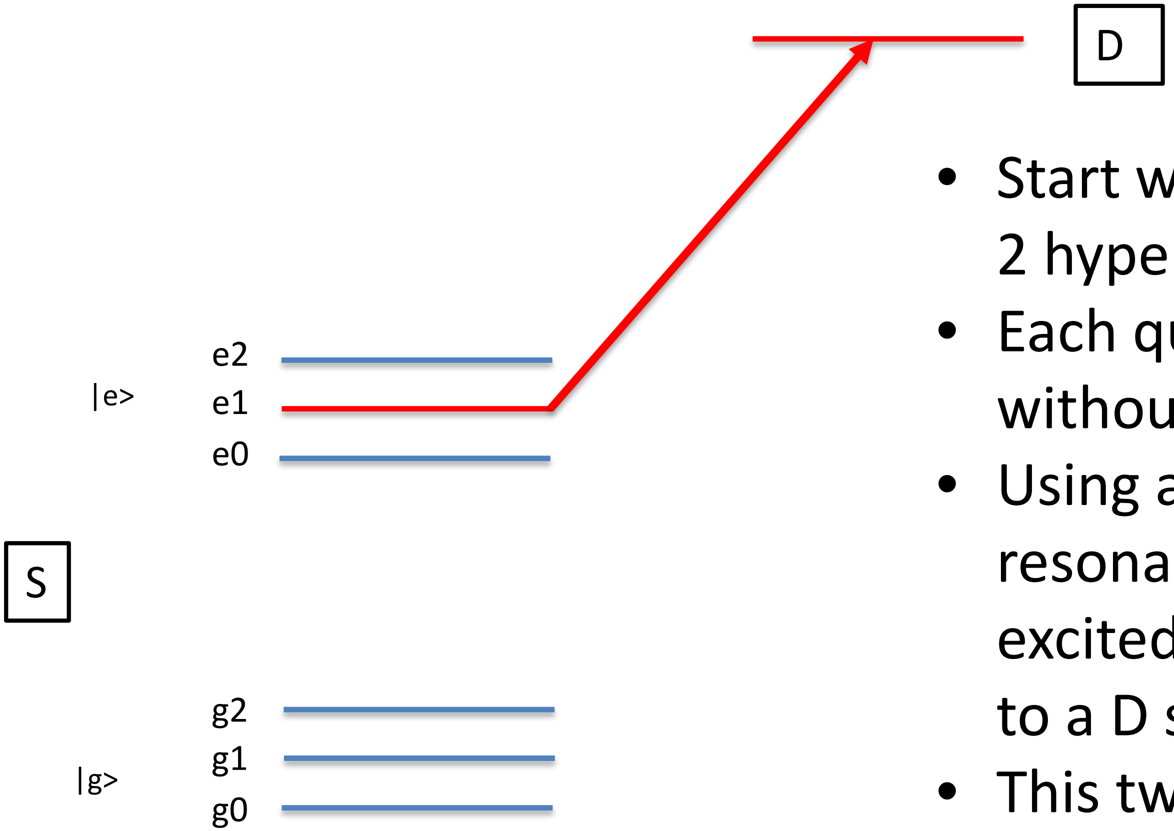
Construct a Phase-Flip Gate

Tuned to frequency to cause transition between |20> and |11> states

Experimental apparatus near absolute zero produces Hilbert Space consisting only of the states listed

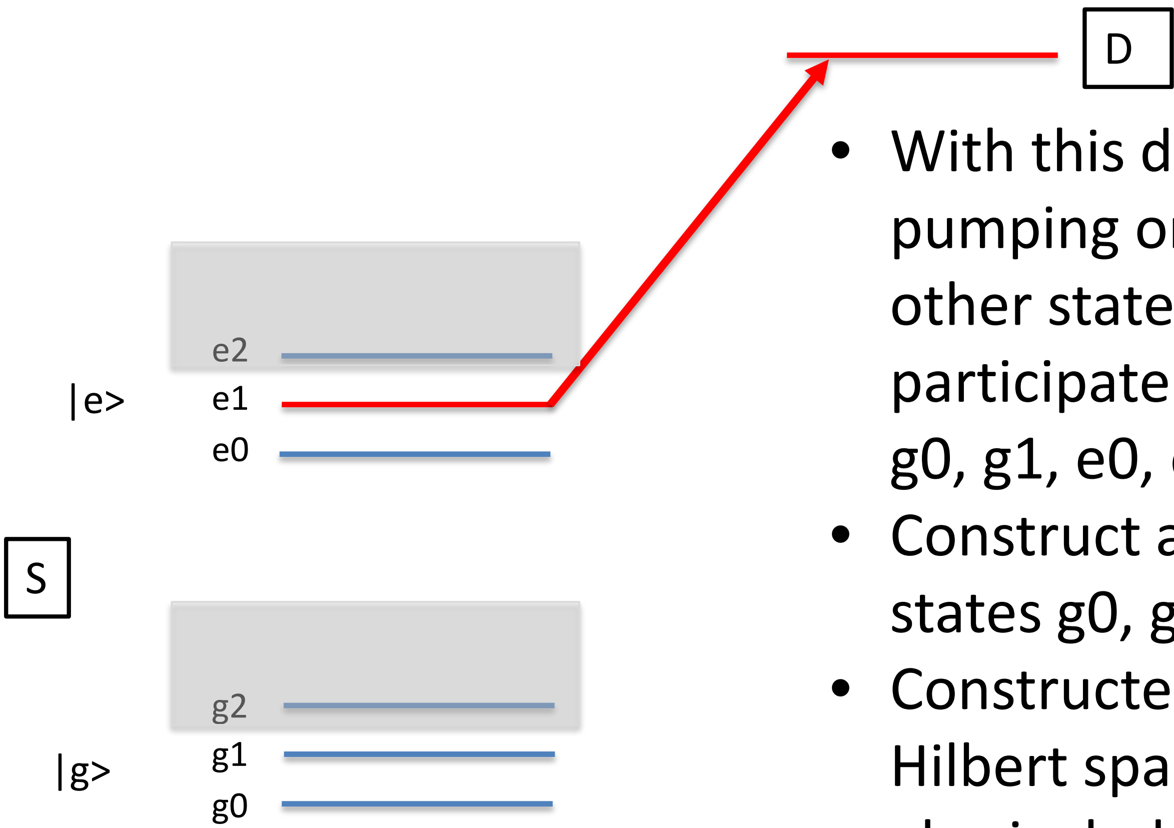


Single Ion Behavior



- Start with ion in an S state with 2 hyperfine states
- Each qubit has |g> and |e> without center of mass motion
- Using a laser can select resonance between the |e1> excited vibrational state of |e> to a D state
- This two state laser driven pulse produces Rabi oscillations

Construct a Phase Gate From Single Ion



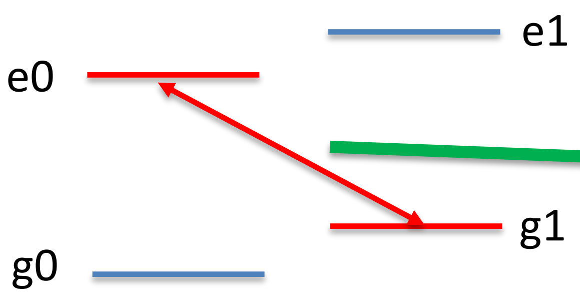
- With this driven laser pulse pumping only this transition the other states that can potentially participate in the interaction are g0, g1, e0, e1
- Construct a Hilbert space with states g0, g1, e0, e1
- Constructed a phase gate in Hilbert space from one ion which also includes the vibrational states of the ion chain center of mass motion

Procedure

- The center of mass (CofM) motion is uniformly detected everywhere along the ion chain
- The 1st qubit recognizes the phase shift and the 2nd qubit has this information encoded into the CofM motion
- Use the CofM motion to construct a phase gate with a ion “A” selected remotely from ion “B” in the ion chain
- When ion “B” is spin up, perform a swapping of the qubit information from the vibrational states in the chain (data bus) using the controlled phase gate on ion “A”
- This swap the ground/excited qubit information into ion “A”

Information Transfer Between Ion Spin State and the Phonon CofM Vibrational State

- Fire another π pulse this time between states |e0> and |g1>
- Transfer the population from |e0> to |g1>



$$(\alpha|g> +\beta|e>)|0> = \alpha|g0> +\beta|e0>$$

$$= \alpha|g0> +\beta|g1>$$

$$\alpha|g0> +\beta|g1> = |g> (\alpha|0> +\beta|1>)$$

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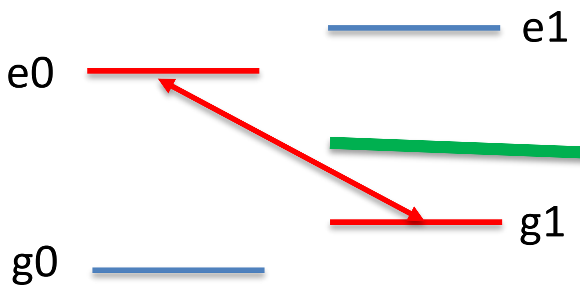


$$(\alpha|g> + \beta|e>)|0> = \alpha|g0> + \beta|e0>$$

$$= \alpha|g0> + \beta|g1>$$

$$\alpha|g0> + \beta|g1> = |g> (\alpha|0> + \beta|1>)$$

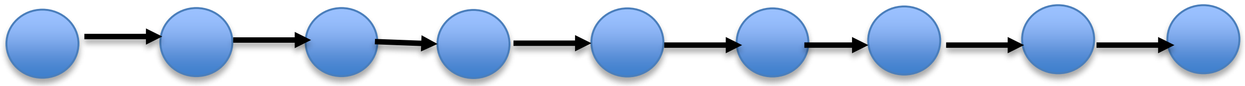
- Probability amplitudes α and β transferred from the internal spin state of the ion to the phonon vibrational center of mass state

Summary of the Operational Steps For the TIQC

- Construct an Initial State
- Transfer information to the data bus (center of mass motion of the phonons in the ion chain)
- Execute a controlled phase gate
- Put the information from data bus back into control ion
- Now the data bus is back in the initial state and ready for the next operation

Can Now Construct Quantum Computer

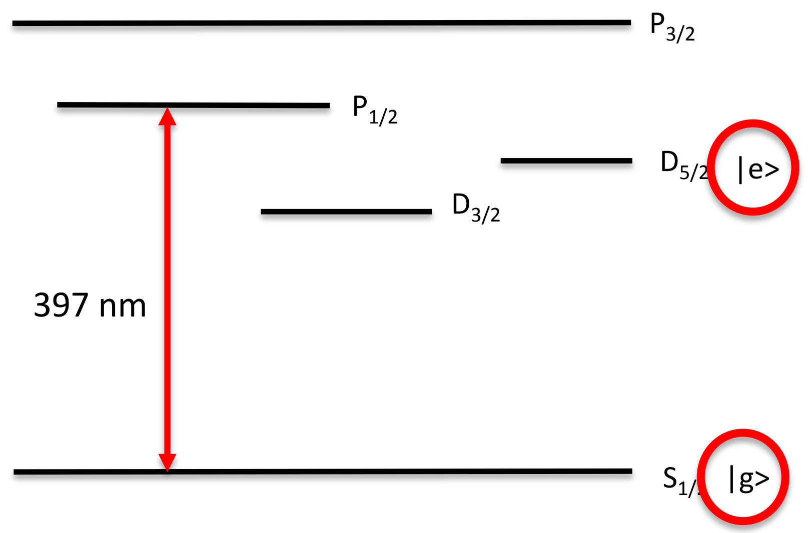
- These 2 ions form quantum computing 2 qubit operations
- Can now operate a TIQC with many ions that provide 2^N states using combinations or any 2 ions remotely separated from each other



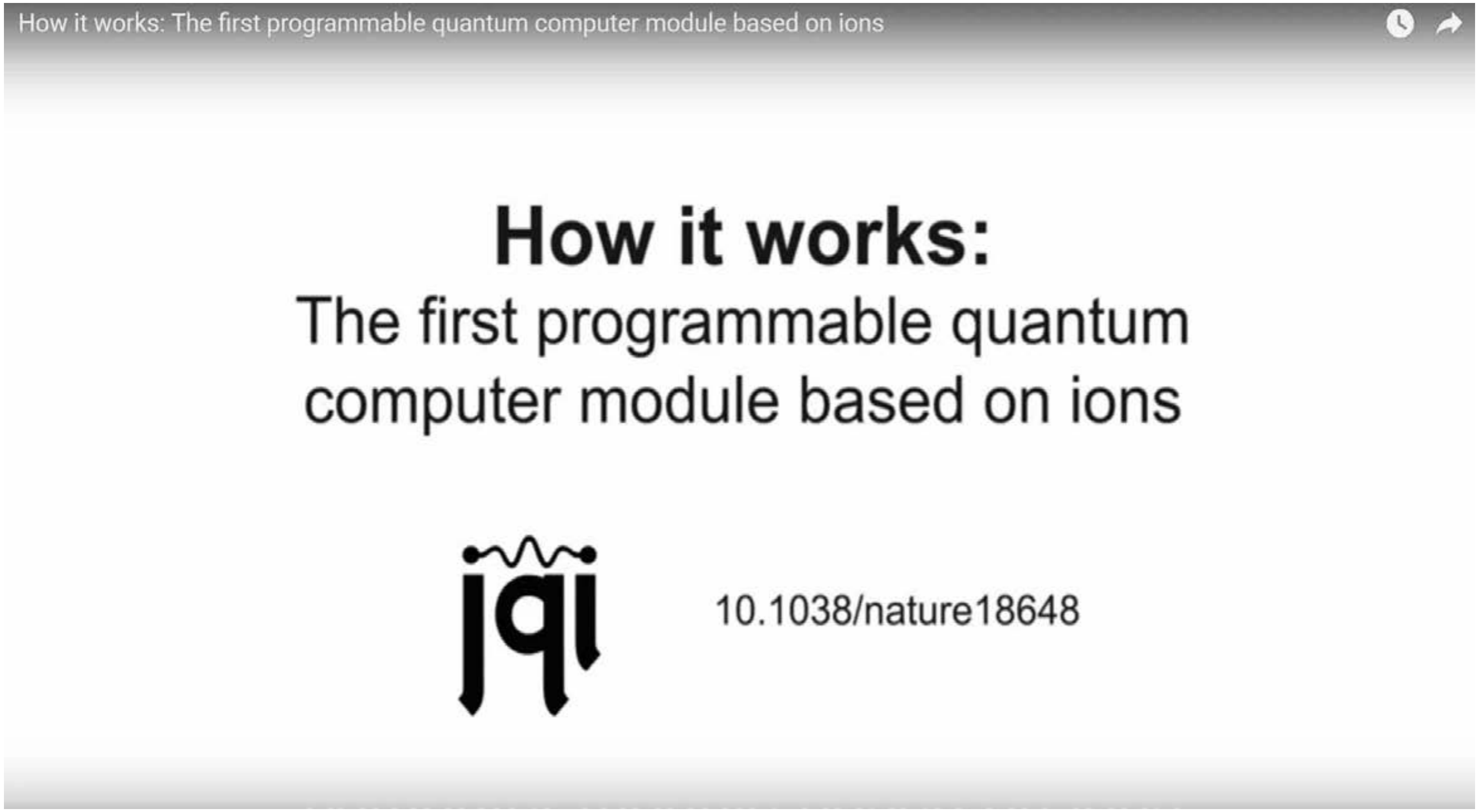
- This procedure selects only the 2 ions that participate in the interaction while all other ions in the chain are undisturbed (no measurement disturbance of the wavefunction)

Measuring the Final State of the Two Qubit System for the $^{40}Ca^+$ Trapped Ion Quantum Computer

- Measurement is done using the 397 nm laser to detect whether or not there is fluorescence between the $P_{1/2} \rightarrow S_{1/2}$ transition
- If the ion is in the ground state ("0" state) then the ion will fluoresce and a 397 nm light signal will be observed
- If the ion is in the $D_{5/2}$ state ("1" state) there will be no fluoresce at 397 nm and no light signal will be observed

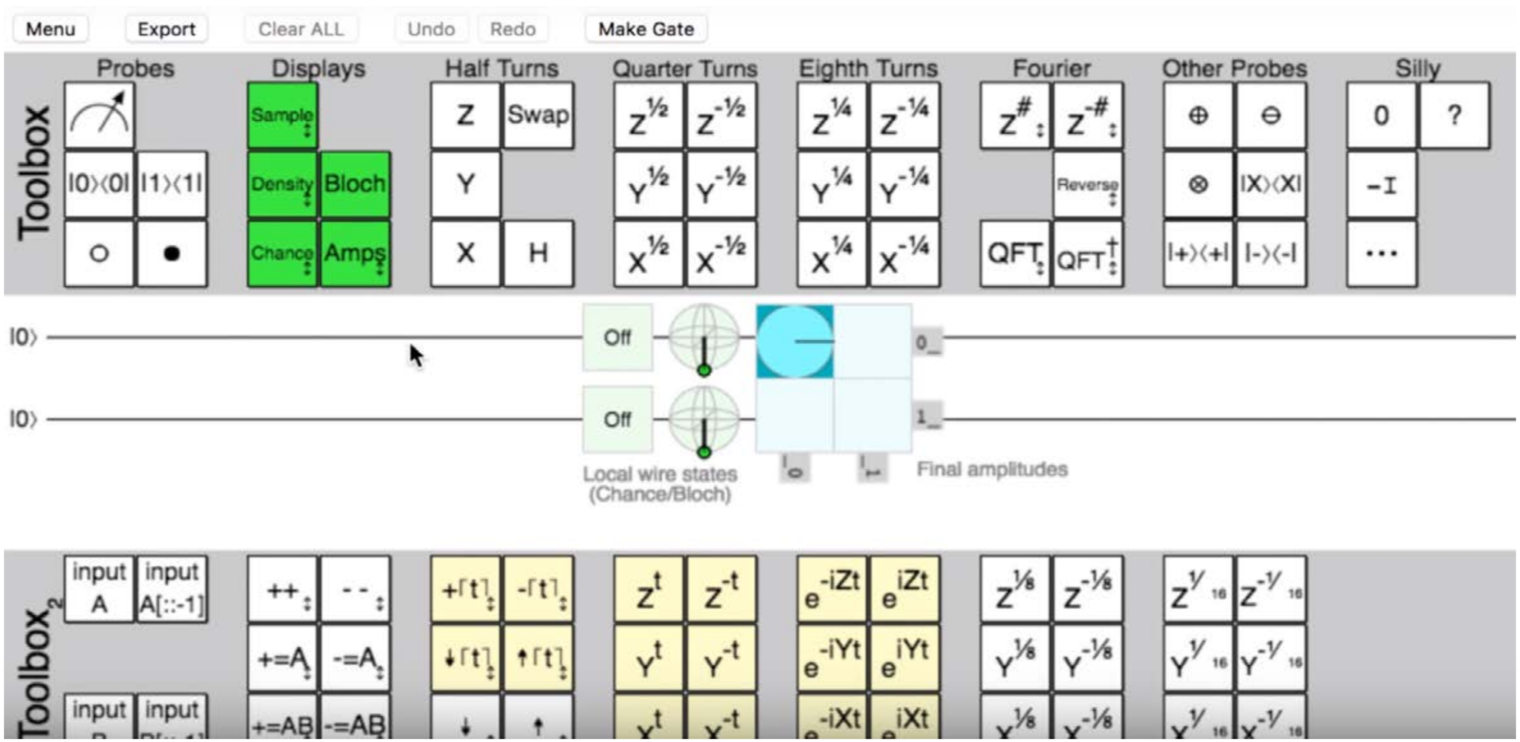


Ion Trap Quantum Computer Simulation



Implement Software With Understanding of 1 Qubit Rotations and 2 Qubit Gates

- Can now use circuit simulator such as Quirk to program a Trapped Ion Quantum Computer



Criteria 5 - System Must Be Scalable

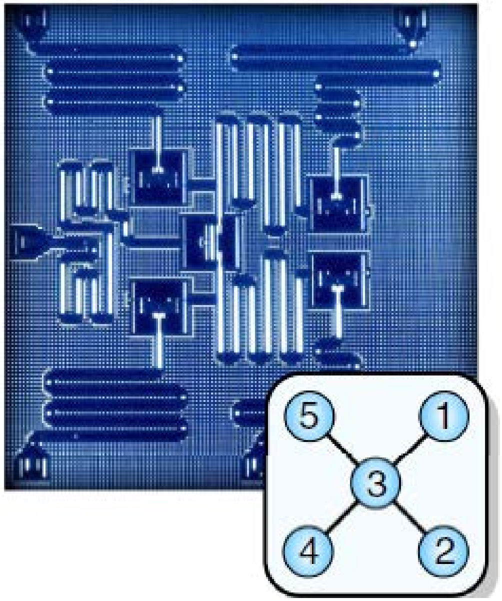
Comments - Ongoing Research
Criteria 5 - System Must Be Scalable

- TIQC requires very pure state initialized which implies very low (milli-Kelvin) operating temperature for the apparatus
- The frequency of the data bus must be slower than the frequency of the center of mass phonon vibrational mode
- As the number of ions increases the difficulty of maintaining a coherent state wavefunction also increases (ex. stray external EM fields) – increasing likelihood of a destroying the coherence and leaving a collapsed wavefunction before the completion of the full set of gate operations

Quantum Computer Architectures Performance Measurements

IBM Quantum Experience Hardware Performance Measurements

- The qubits are connected in a star-shaped pattern that provides four 2-qubit interactions which are controlled-NOT (CNOT) gates targeting the central qubit. Single-qubit readout fidelities are typically 96% (www.research.ibm.com/ibm-q),



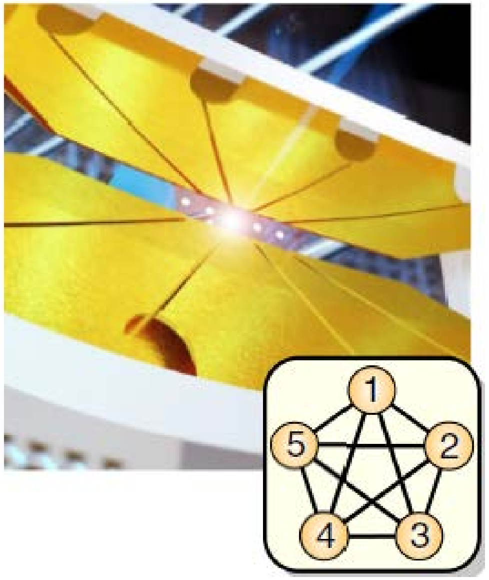
IBM Quantum Experience Hardware Performance Measurements*

- Average readout fidelity for an arbitrary 5-qubit state is 80% (www.research.ibm.com/ibm-q)
- Typical gate fidelities are 99:7% and 96:5% for single- and 2-qubit gates
- Typical gate times are approximately 130 ns for single-qubit gates and 250 – 450 ns for 2-qubit gates
- Coherence times are ~ 60 μs for both depolarization (T1) and spin dephasing (T2)

* Experimental comparison of two quantum computing architectures, Norbert M. Linke, Dmitri Maslov, Martin Roetteler, Shantanu Debnath, Caroline Figgatt, Kevin A. Landsman, Kenneth Wright and Christopher Monroe, PNAS, March 28, 2017, vol. **114** no. 13, 3305–3310

Trapped Ion Quantum Computer Performance Measurements

- The qubits are connected in a star-shaped pattern that provides pairwise 2-qubit interactions among all qubits



Ion Trap Hardware Performance Measurements*

- The addressing during operations and the distinction between qubits during readout are both achieved by spatially resolving the ions.
- The fidelities for single- and 2-qubit gates are typically 99.1(5)% and 97(1)%, respectively.
- The single-qubit readout fidelity is 99:7(1)% for state |0> and 99.1(1)% for state |1> (The latter is lower because off-resonant excitation during readout predominantly causes |1> → |0> pumping)
- The average readout fidelity for an entire 5-qubit state is 95.7(1)%.
- This is lower than one would expect from the average single-qubit readout fidelity, because there is crosstalk that leads to |1> → |0> errors on adjacent channels.

* Performance data courtesy of Univ of Maryland Joint Center for Quantum Information and Computer Science

Ion Trap Hardware Performance Measurements*

- Typical gate times are 20μs for single-cubit and 250μs for 2-qubit gates.
- Spin depolarization is negligible for hyperfine ground-level qubits ($T_1 \sim \infty$)
- The spin-dephasing time (T_2^*) is ~ 0.5 s in the current setup and can be easily extended by suppressing magnetic-field noise.

* Performance data courtesy of Univ of Maryland Joint Center for Quantum Information and Computer Science

Summary – Operation of the TIQC

- General quantum algorithm can be constructed using
  - single qubit rotations (oscillations on the Bloch sphere from ground to excited state plus all superposition states in between)
  - 2 qubit gates
- Can now use circuit simulator software such as Quirk to build programs that can be loaded and run in a TIQC

Quirk tutorial product warning label

- Need to understand the quantum properties of the 1 qubit rotations and 2 qubit gates otherwise likely to construct arbitrary mixtures of 1 and 2 qubit gates that will run but are meaningless because they don't map the Quantum Computer to the physical problem to be simulated

Return to one of the slides from my Part I Lecture

Why is it Hard to Build Quantum Computers and Develop QC Algorithms

- Problem 1
  - If one wants to use quantum mechanics to build a computer, one must understand workings of the quantum world to know how a quantum computer will process a problem
  - However
    - All human experiences rooted in the classical world
    - Human experience and intuition will tend to think of ideas approaches that are biased toward past experiences and expected behaviors
    - Quantum computers behave in ways that have no classical analog
    - There is no prior direct human experience on which to rely for intuition
- Problem 2
  - Even if an algorithm or program can be shown to be based on quantum mechanical systems it must be demonstrated that the quantum mechanical algorithm is better than the classical equivalent

Last Slide - PART V