Principle Concepts behind IBM Q

D. Stancil, Quantum Computing Seminar, 15 Feb 2018

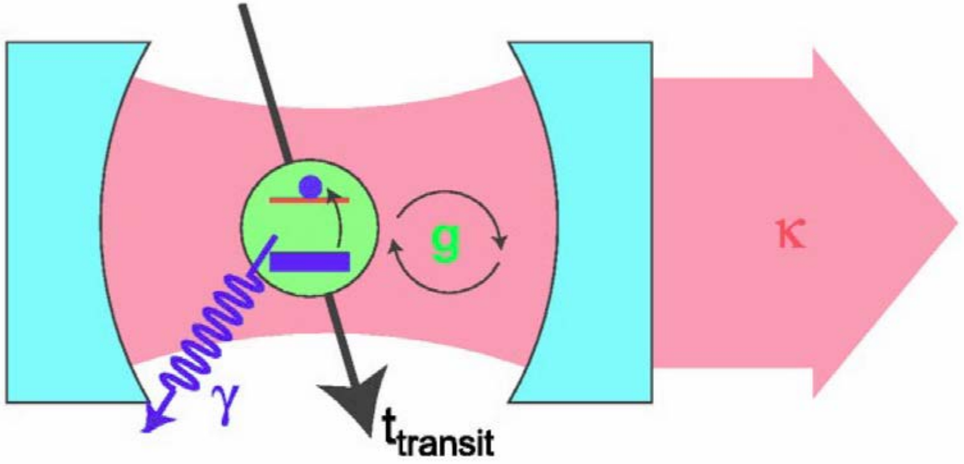
- V. Bouchiat, et al, “Quantum Coherence with a Single Cooper Pair,” Physica Scripta, Vol. T76, 165-170 (1998)
- A. Blais, et al, “Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation,” Phys. Rev. A, Vol. 9, 062320 (2004)
- J. Koch, et al, “Charge-insensitive qubit design derived from the Cooper Pair Box,” Phys. Rev. A Vol. 76, 042319 (2007).
- J. Clarke & F. Wilhelm, “Superconducting quantum bits,” Nature, Vol. 453, 19 June 2008, p. 1031.
- J. Gambetta, et al, “Building logical qubits in a superconducting quantum computing system,” npj Quantum Information, 13 Jan. 2017

Outline

- Cavity Quantum Electrodynamics
- Rabi oscillations
- Cooper pairs and superconductivity
- Josephson junction
- SQUID
- Cooper Pair Box
- Transmon
- Entangled transmons
- IBM examples

Cavity Quantum Electrodynamics

- EM field creation & annihilation operators: $a^{\dagger}, a$
- Atomic energy level raising and lowering operators: σ^{+}, σ^{-}
- Hamiltonian of coupled system (Jaynes-Cummings Hamiltonian):



Blais, et al

$$H = \underbrace{\hbar\omega_r\left(a^{\dagger}a + \frac{1}{2}\right)}_{\text{EM field}} + \underbrace{\frac{\hbar\Omega}{2}\sigma^z}_{\text{atomic hamiltonian}} + \underbrace{\hbar g\left(a^{\dagger}\sigma^{-} + \sigma^{+}a\right)}_{\text{Coupling between atom and EM field}} + \underbrace{H_{\kappa}}_{\text{Cavity energy loss}} + \underbrace{H_{\gamma}}_{\text{Radiative energy loss}}$$

Rabi Oscillations

- When a two-level system is coupled to a driving field at precisely the frequency corresponding to the energy difference between the states, the system will oscillate between the two states at the Rabi frequency

$$|\psi\rangle = c_0(t)|0\rangle + c_1(t)|1\rangle$$

if $c_0(0)=0,\ c_1(0)=1,$ then

$$P_0(t) = |c_0(t)|^2 = \frac{1}{2}\left(1-\cos\Omega_R t\right)$$

$$P_1(t) = |c_1(t)|^2 = \frac{1}{2}\left(1+\cos\Omega_R t\right)$$

$$\Omega_R = \frac{2}{\hbar}\langle 1|H_I|0\rangle$$

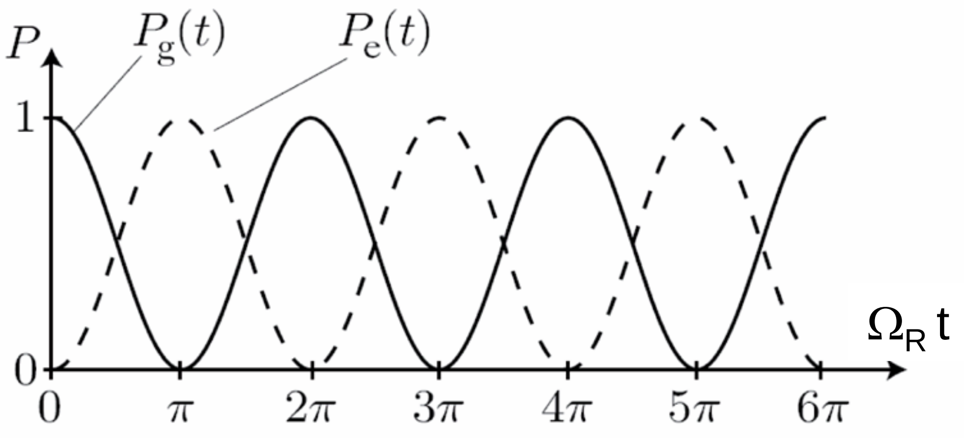


Figure 51: Time evolution of the probability $P_g(t)$ and $P_e(t)$ to find the atom in the ground (solid) and excited (dashed) state, respectively. [from D.A. Steck *Quantum and Atom Optics*]

Changing States with Pulses

- "π-pulse": $\Omega_R t = \pi$ inverts the state
- "π/2-pulse": $\Omega_R t = \pi / 2$ creates equal superposition of states (Hadamard gate)
- Key point: you can flip a state or create a superposition state by controlling the pulse length

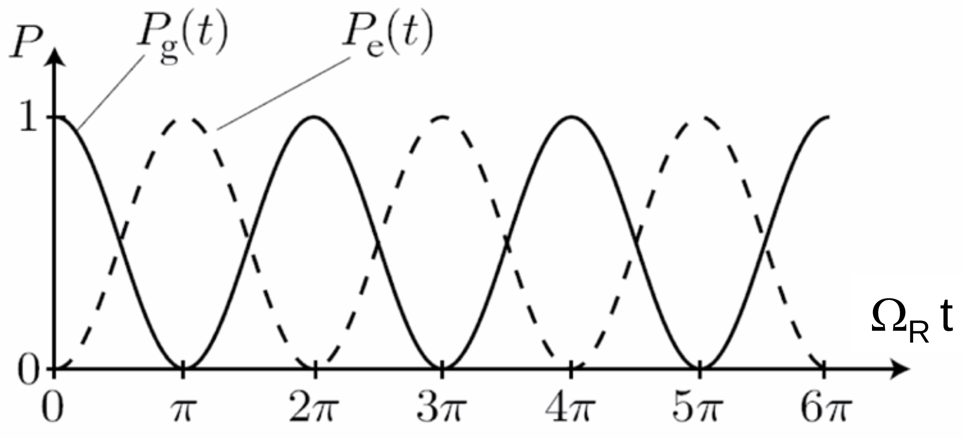


Figure 51: Time evolution of the probability $P_g(t)$ and $P_e(t)$ to find the atom in the ground (solid) and excited (dashed) state, respectively. [from D.A. Steck *Quantum and Atom Optics*]

Cooper Pairs and Superconductivity

- Spin ½ particles are "Fermions"
  - Fermions obey the Pauli exclusion principle: no two can be in the same state
  - Electrons are Fermions
- Spin 1 particles are "Bosons"
  - Bosons do not obey the Pauli exclusion principle: you can have as many in a state as you want
  - Photons are Bosons
- In a superconductor, an effective attractive interaction between electrons causes them to be loosely bound together and act like a single spin 1 particle: "Cooper Pair"
- Since Cooper pairs are spin 1, they act like Bosons, and you can have multiple Cooper pairs in the same state

Cooper Pairs are the result of the Electron-Phonon interaction in the theory of Bardeen, Cooper, and Schreifer (BCS Theory)

- Electrons normally repel one another, but are attracted to ions in the crystal lattice
- If the ions are pulled slightly toward an electron, from a distance it can appear as though there is a net positive charge, attracting another electron

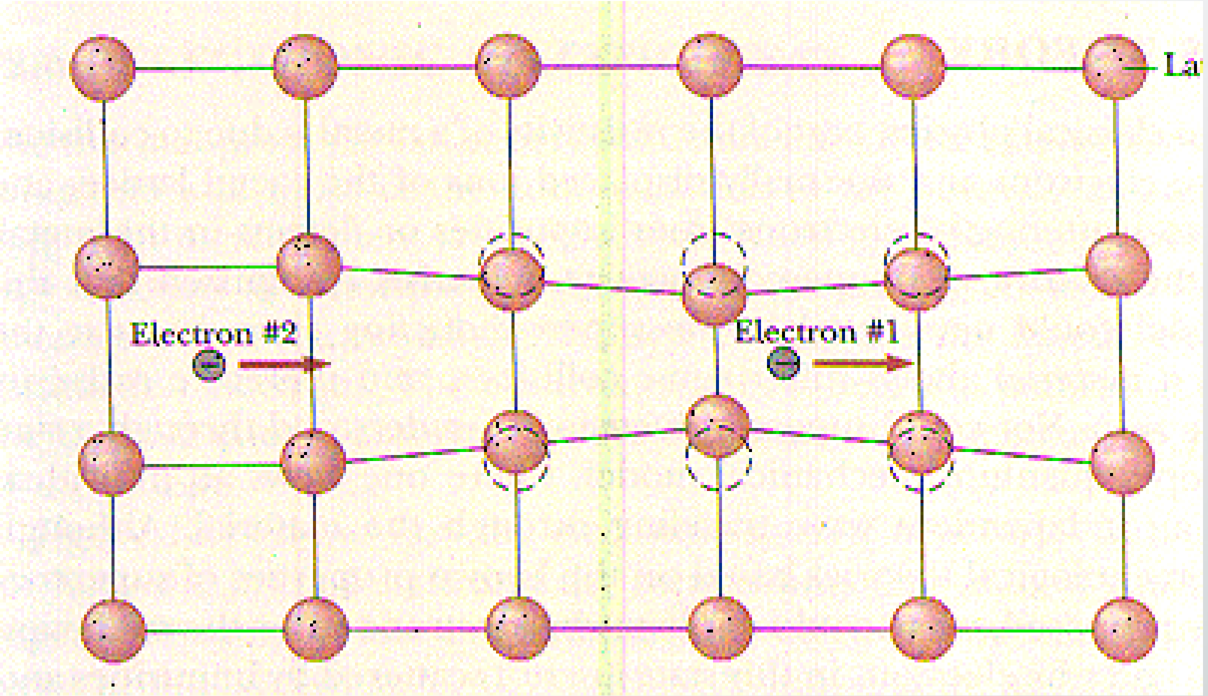
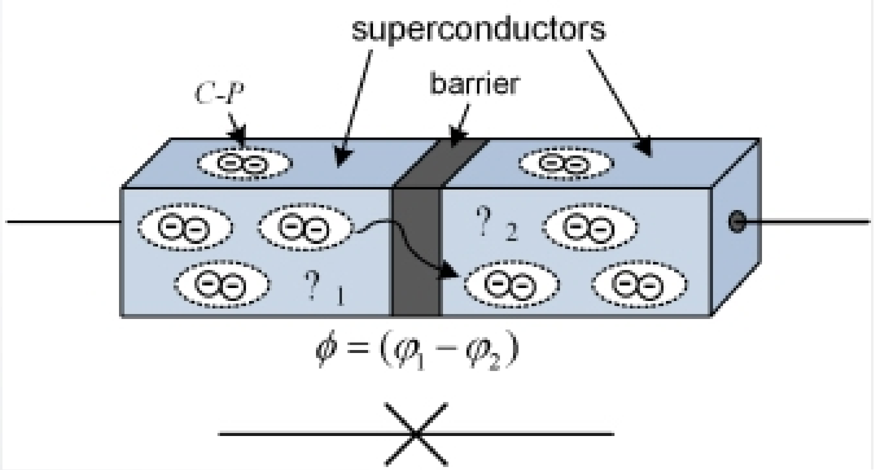
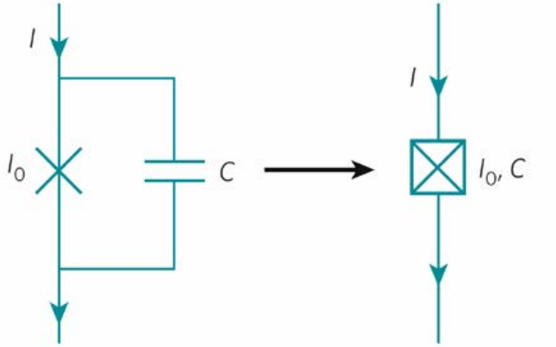


Image from Quora

Josephson tunnel junction





$$I = I_c \sin\varphi$$

$$V = \frac{\Phi_0}{2\pi}\frac{d\varphi}{dt}$$

Clarke & Wilhelm

$$\frac{dI}{dt} = I_c \cos\varphi \frac{d\varphi}{dt} = I_c \cos\varphi \frac{2\pi V}{\Phi_0}$$

$$\Rightarrow$$

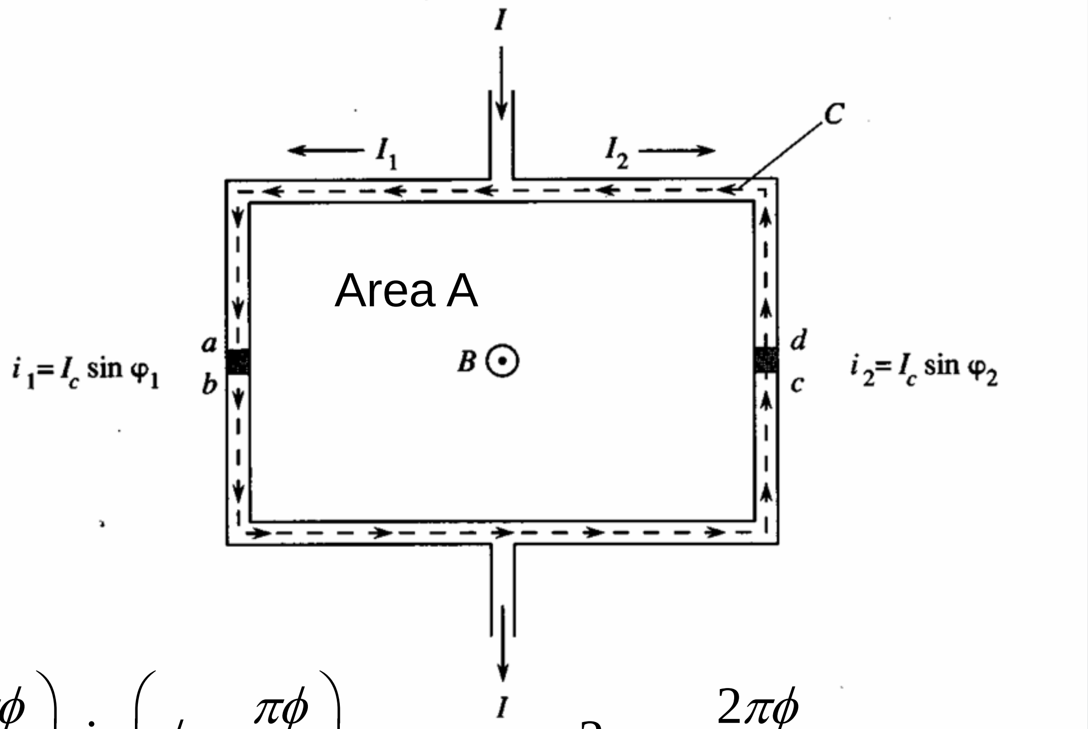
$$V = \frac{\Phi_0}{2\pi I_c \cos\varphi}\frac{dI}{dt} = \frac{\Phi_0}{2\pi I_c \sqrt{1-\sin^2\varphi}}\frac{dI}{dt} = \frac{\Phi_0}{2\pi I_c \sqrt{1-(I/I_c)^2}}\frac{dI}{dt} = L_{eff}(I)\frac{dI}{dt}$$

- Looks like a non-linear inductor: origin of *anharmonicity*: spacing between energy levels is not the same
  - *Enables the individual addressing of a single pair of states*
  - *In contrast, in the harmonic oscillator, all states are equally spaced*

Ph.D. thesis: Vratislav Michal

Superconducting Quantum Interference Device (SQUID)

- Parallel Josephson Junctions
- Current depends on applied magnetic field
- Preview: enables qubits to be “tuned” by an external magnetic field



$$i = 2I_c \cos\left(\frac{\pi\phi}{\Phi_0}\right)\sin\left(\phi_1 + \frac{\pi\phi}{\Phi_0}\right), \quad \varphi_2 - \varphi_1 = 2\pi n + \frac{2\pi\phi}{\Phi_0}$$

$$\phi = \phi_{ext} + \frac{LI_c}{2}\sin\left(\frac{\pi\phi}{\Phi_0}\right)\cos\left(\phi_1 + \frac{\pi\phi}{\Phi_0}\right)$$

(transcendental equation for ϕ)

$$\phi_{ext} = BA$$

MIT 6.763 2003 Lecture 12

Cooper Pair Box

- Energy in a capacitor:

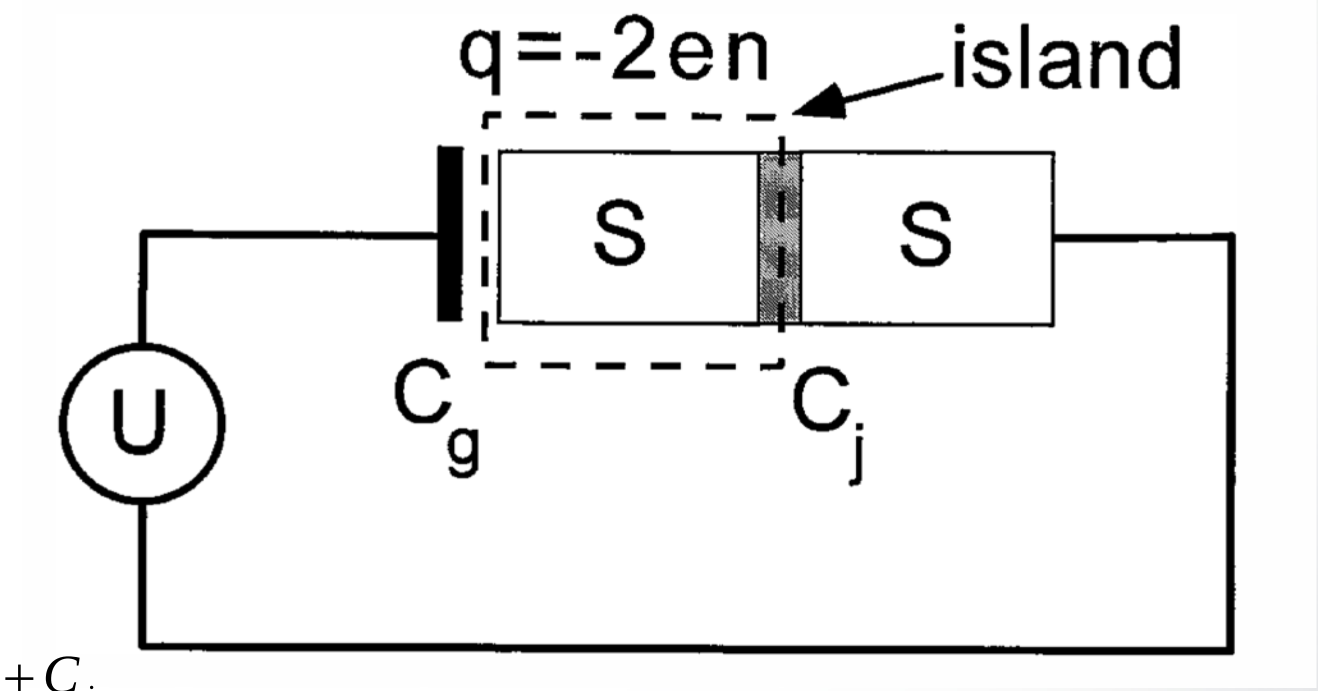
$$E_c = \frac{1}{2}CU^2, Q = CU$$

$$\Rightarrow E_c = \frac{Q^2}{2C}$$

- For an electron:

$$Q = -e,$$

$$\Rightarrow E_c = \frac{e^2}{2C_\Sigma},\ C_\Sigma = C_g + C_j$$



- E_c is the “charge energy” associated with a single electron
- Total charge energy with n C-Ps: $Q = -2en$

$$E_q = 4E_c n^2$$

Bouchiat, et al

Cooper Pair Box Hamiltonion

- The charge stored on a box with an applied voltage U to the “gate”:

$$Q = C_g U = -2en_g$$

$$n_g = -\frac{C_g U}{2e}$$

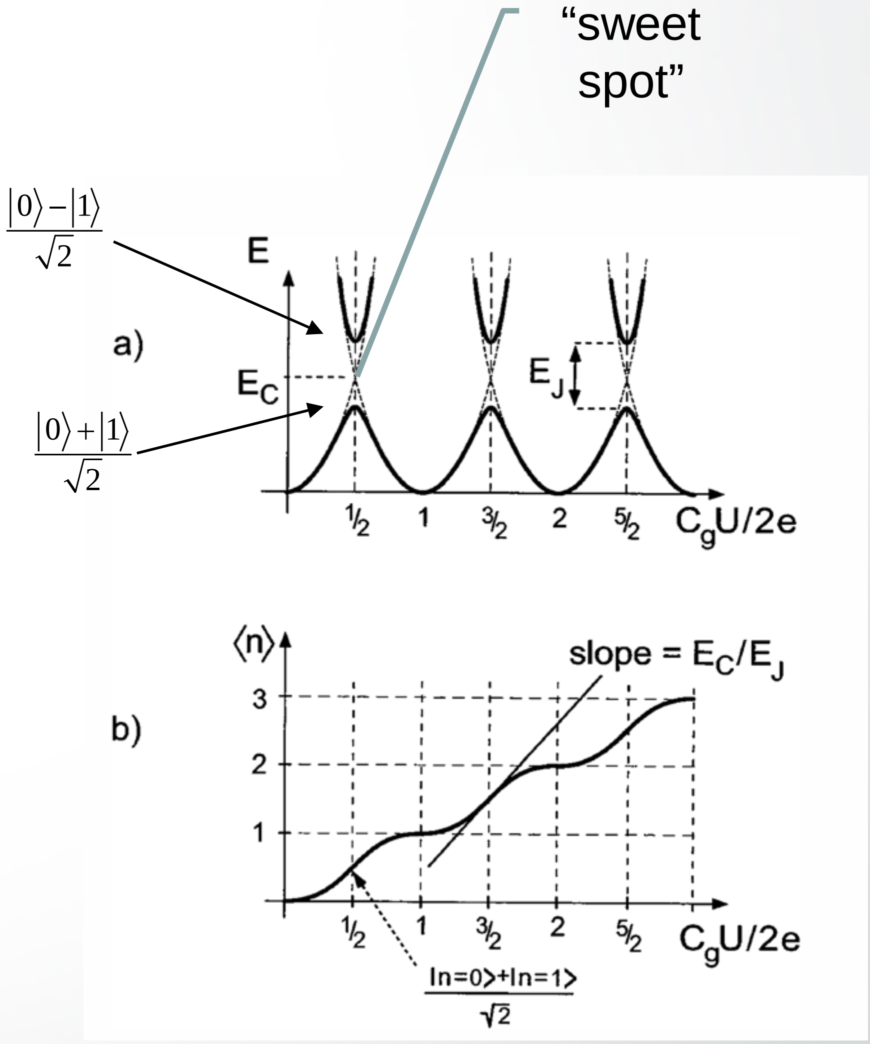
- The Hamiltonian of an isolated box can be written

$$H_Q = 4E_c \sum_n \left(n - n_g\right)^2 |n\rangle\langle n|$$

- Additional C-Ps can tunnel in & out through the Josephson junction, described by the coupling Hamiltonian

$$H_J = -\frac{E_J}{2} \sum_n \left(|n\rangle\langle n+1| + |n+1\rangle\langle n|\right)$$

- This interaction opens “gaps” of width E_J at the crossing points



Bouchiat, et al.

Two lowest states of CPB

$$H|\psi\rangle = E|\psi\rangle$$

$$\left[4E_c\sum_n (n-n_g)^2 |n\rangle\langle n| - \frac{E_J}{2}\sum_n \left(|n\rangle\langle n+1| + |n+1\rangle\langle n|\right)\right]|\psi\rangle = E|\psi\rangle$$

Consider the lowest two states only, $|0\rangle, |1\rangle$:

$$4E_c n_g^2|0\rangle - \frac{E_J}{2}|1\rangle = E|0\rangle$$

$$4E_c(1-n_g)^2|1\rangle - \frac{E_J}{2}|0\rangle = E|1\rangle$$

$$E_0|0\rangle - \frac{E_J}{2}|1\rangle = E|0\rangle$$

$$E_1|1\rangle - \frac{E_J}{2}|0\rangle = E|1\rangle$$

Look for Eigenvalues and Eigenvectors:

$$\begin{bmatrix} E_0 & -E_J/2 \\ -E_J/2 & E_1 \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = E\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

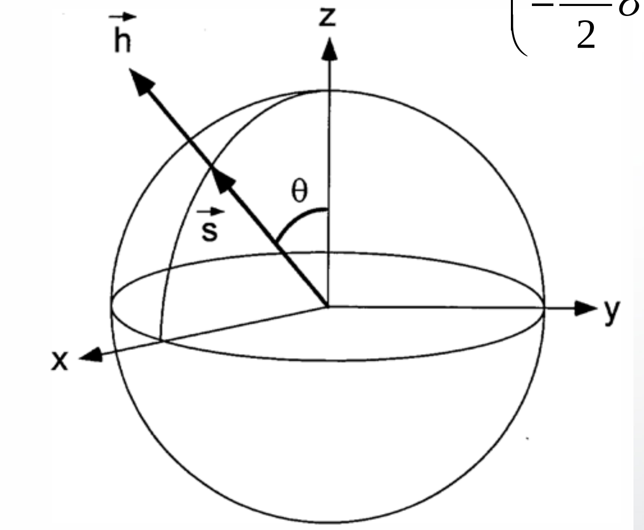
$$\begin{bmatrix} -E_\Delta/2 & -E_J/2 \\ -E_J/2 & E_\Delta/2 \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = E'\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

$$\left(-\frac{E_\Delta}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - \frac{E_J}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right)\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = E'\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

$$\left(-\frac{E_\Delta}{2}\sigma^x - \frac{E_J}{2}\sigma^z\right)\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = E'\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

$$\Rightarrow H = -\mathbf{s}\bullet\mathbf{h},$$

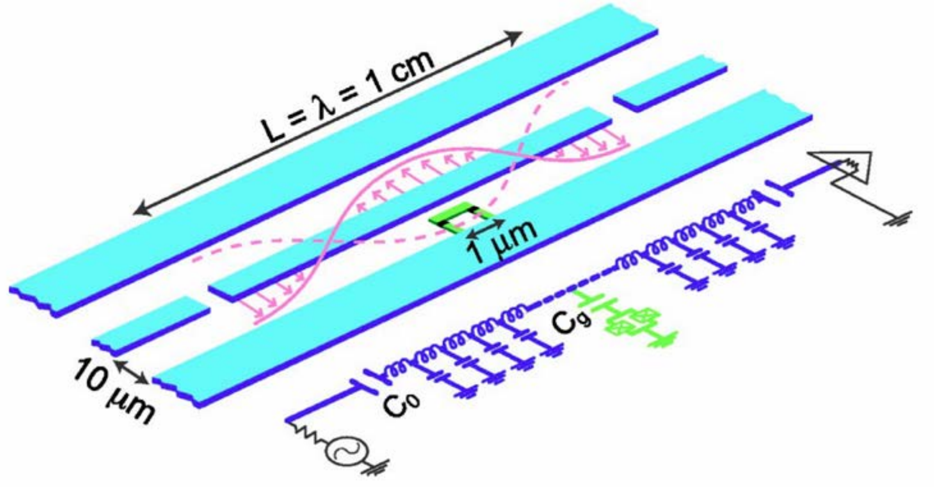
$$\mathbf{s} = \frac{1}{2}\vec{\sigma},\ \mathbf{h} = \hat{x}E_J + \hat{z}E_\Delta$$



Bouchiat, et al.

Circuit Quantum Electrodynamics

- Co-planar microstrip resonator formed by gaps in center conductor
- Isomorphic with Cavity Quantum Electrodynamics used in trapped ion quantum computers
- Express Hamiltonian in “up” & “down” states along the “field” h

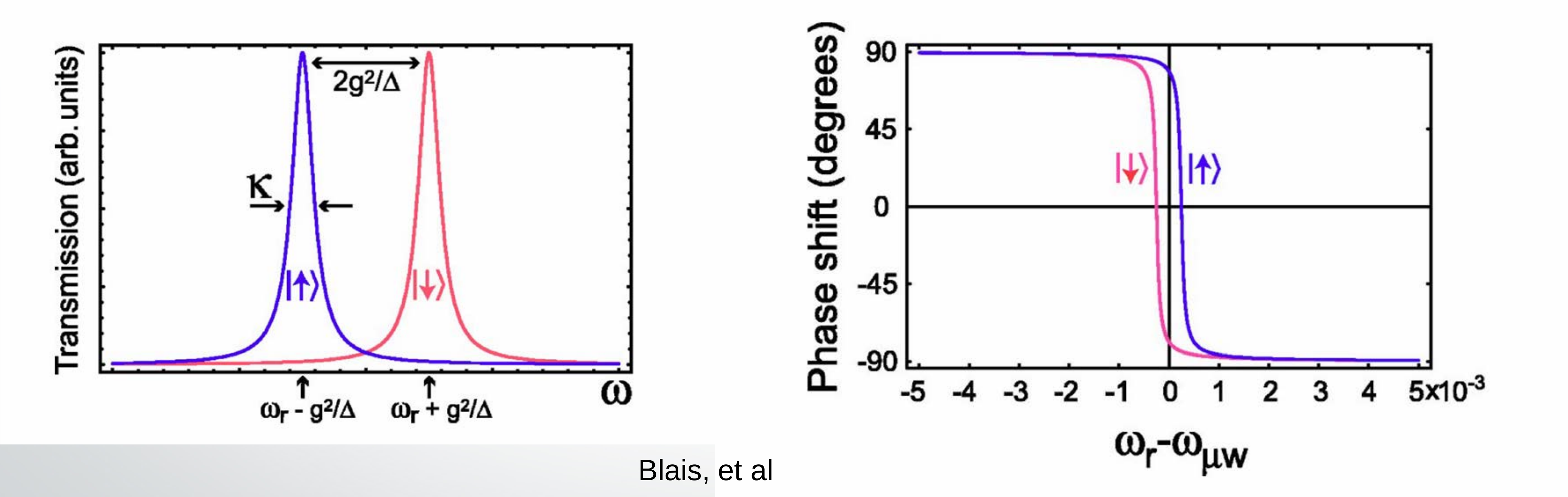


Blais, et al

$$H = \underbrace{\hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)}_{\text{EM field}} + \underbrace{\frac{\hbar\Omega}{2}\sigma^z}_{\text{CPB hamiltonian}} + \underbrace{\hbar g\left(a^\dagger\sigma^- + \sigma^+ a\right)}_{\text{Coupling between CPB and EM field in coplanar transmission line}}$$

Control and Read-out

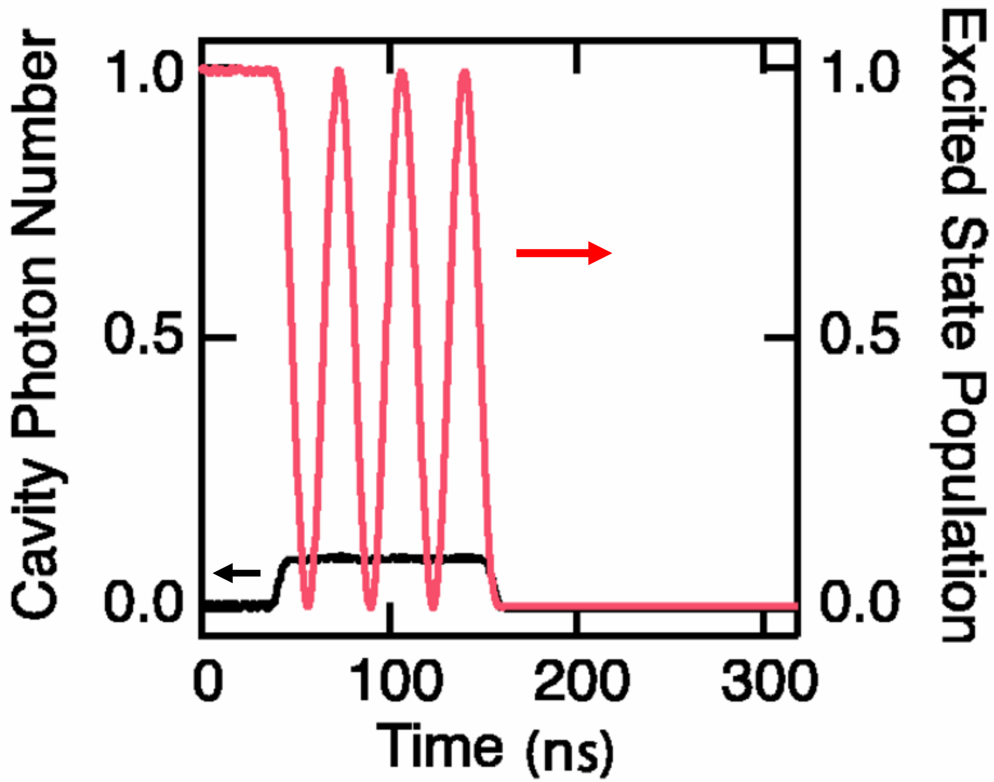
- When the CPB couples to the microstrip resonator and both are tuned to the same frequency, there is a splitting into two modes
- Sending in a pulse near ω_r enables you to read-out the state either from the phase, or the amplitude
- Sending in a pulse detuned from the resonance rotates the state, but does not make a measurement (there is no information about the state in the reflected signal)



Blais, et al

Control Pulse simulation

- Control signal turned on for 7 pi pulses then turned off
- The state rotates between the excited and ground states
- The cavity photon level is small since the cavity is detuned from resonance



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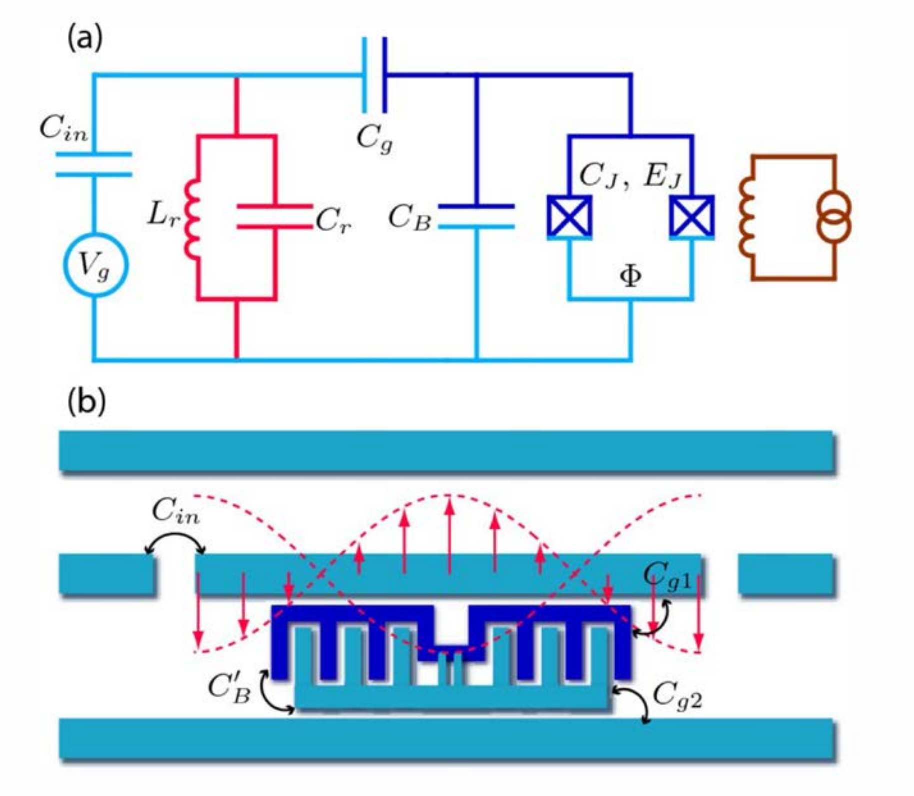
Cooper Pair Box vs Transmon

- Quantities crucial to operation of CPB:
  - Anharmonicity (energy levels not equally spaced—results from JJ nonlinearities)
  - Charge dispersion (variation of energy levels to fluctuations in offset charge and gate voltage—reduces coherence time)
- Key ratio: E_J/E_C
  - Small values give larger anharmonicity and clean CPB charge states
  - Large values reduce anharmonicity but also reduce sensitivity to charge fluctuations
  - Anharmonicity reduces algebraically with increasing E_J/E_C while charge dispersion reduces exponentially with increasing E_J/E_C
- Transmon: similar to CPB but operates at a large E_J/E_C to increase coherence time while maintaining adequate anharmonicity

Transmon

- Adds extra capacitance by adding λ/20 transmission lines to either side of a pair of Josephson Junctions forming a SQUID
  - Increases the ratio E_J/E_c by reducing E_c :

$$E_c = \frac{e^2}{2C_\Sigma}, \; C_\Sigma = C_J + C_g + C_B$$



Koch, et al (not to scale)

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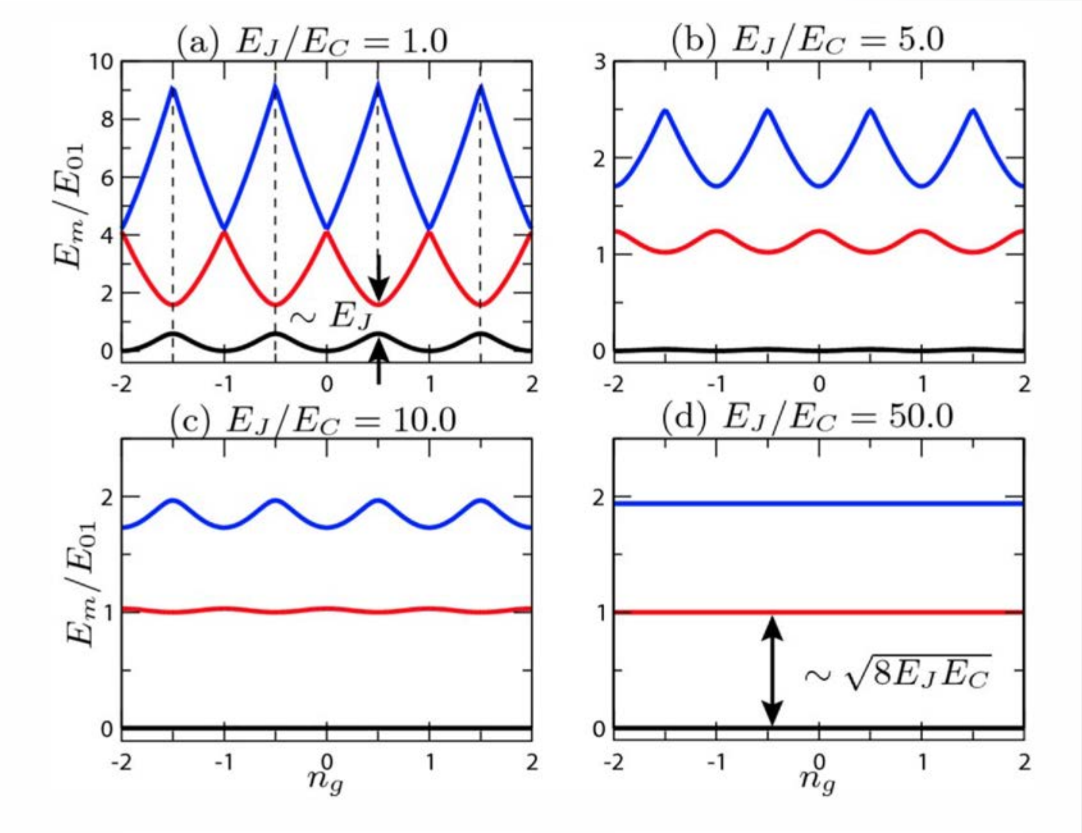
Reducing the Sensitivity to bias voltage with E_J/E_c

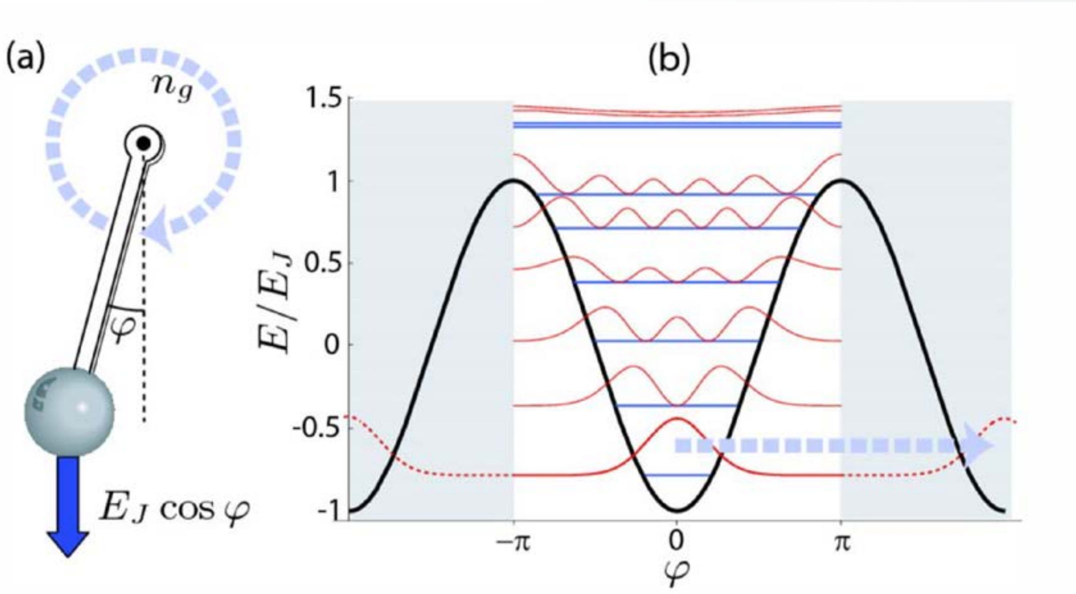
- In the large E_J/E_c limit, the energy difference between the ground and excited states becomes insensitive to the bias voltage
- Sufficient difference in energy levels is maintained



Koch, et al.

Mechanical Analogy

- Large E_J/E_c means gravity dominates the effect of the charge & B field
- Look at excitations near $\varphi = 0$
- Charge noise stability the result of exponential dependence of tunneling on barrier width
- Anharmonicity is the result of the sinusoidal potential energy



$$n \leftrightarrow L_z / \hbar$$

$$E_J = mgl$$

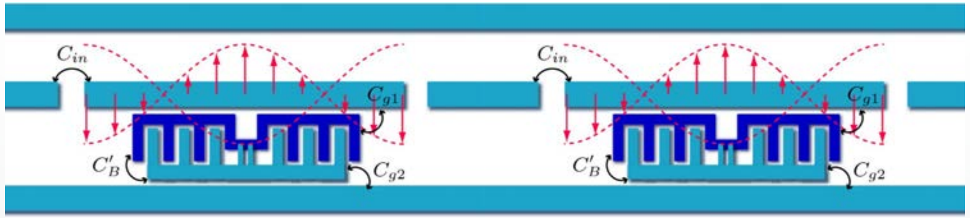
$$E_c = \frac{\hbar^2}{8ml^2}$$

$$n_g = \frac{qB_0 l^2}{2\hbar}$$

Koch, et al.

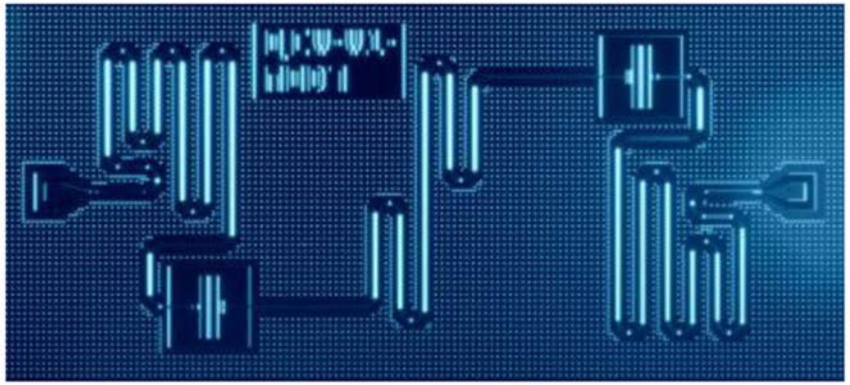
Entangling multiple qubits

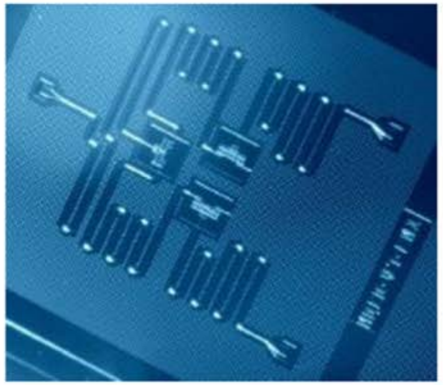
- Qubits coupled through the photon field
- Multiple qubits could be placed in the same co-planar microstrip resonator
- Qubits in separate resonators can be coupled via a microstirp bus



Example Devices Fabricated at IBM

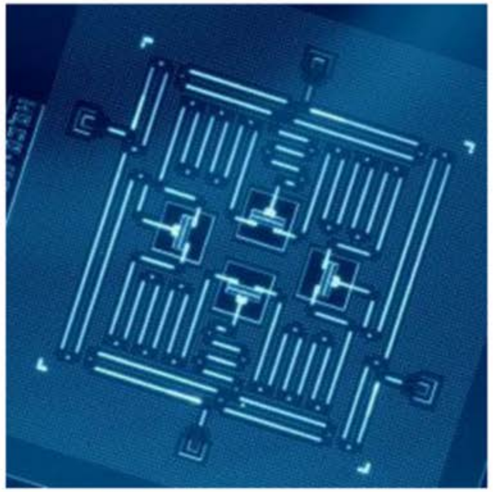
2 qubits
1 bus
2 readout resonators
(2-qubit gates)

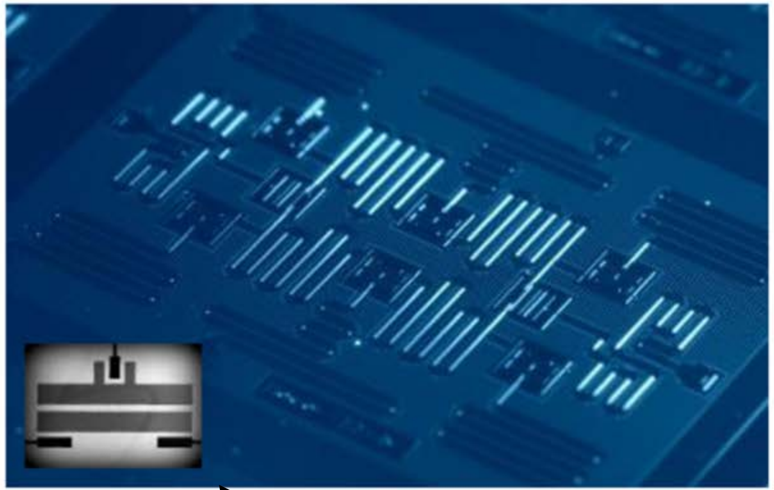




3 qubits
2 bus
3 readout resonators
(demonstrated parity
Measurement)

4 qubits
4 bus
4 readout resonators
(demonstrated
[2,0,2] code)





8 qubits
4 bus
8 readout resonators
(study both Z and X
Parity checks)

Gambetta, et al

Micrograph of individual transmon qubit