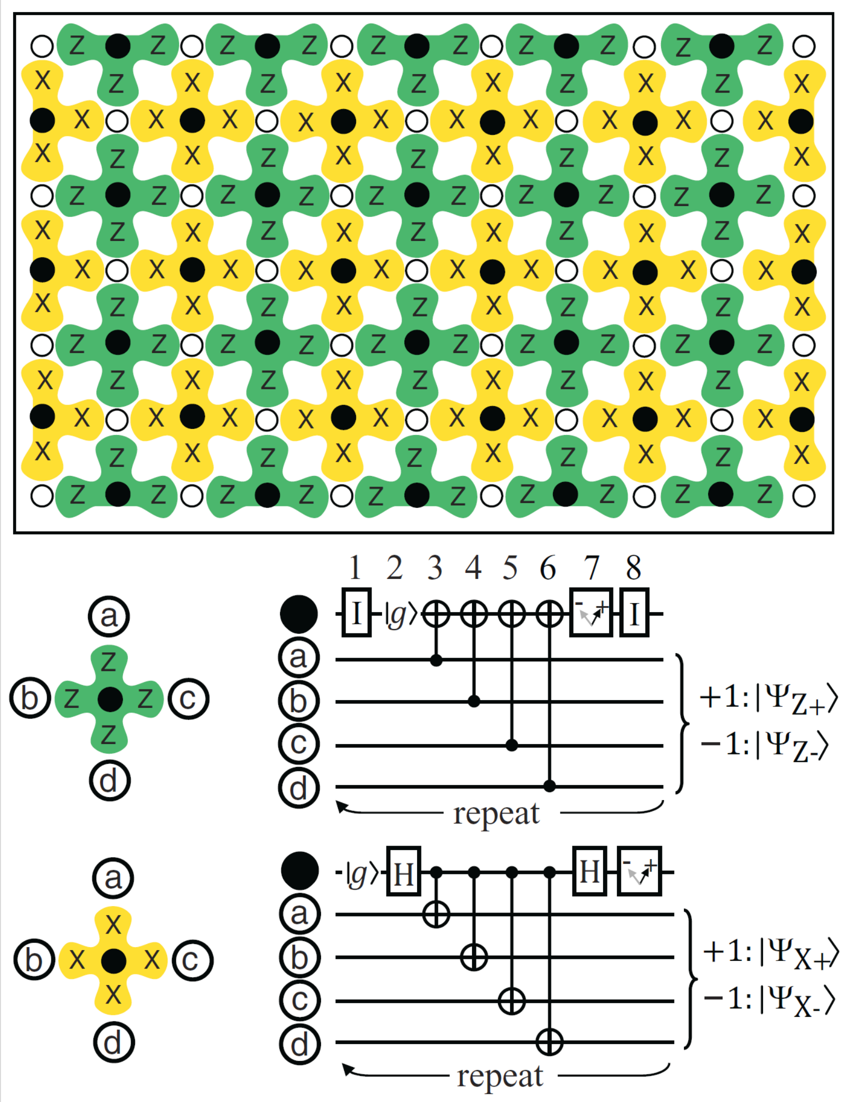


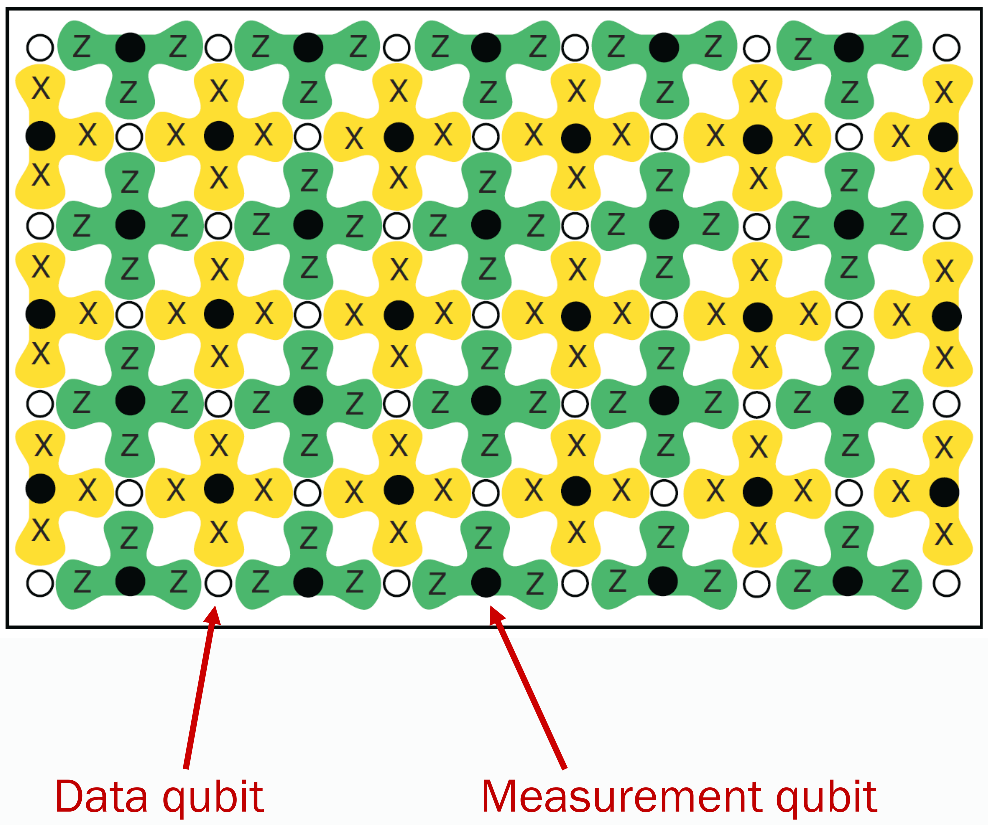
Quantum Error Correction: Part 2
Surface Code

ECE 592/CSC 591 – Fall 2019

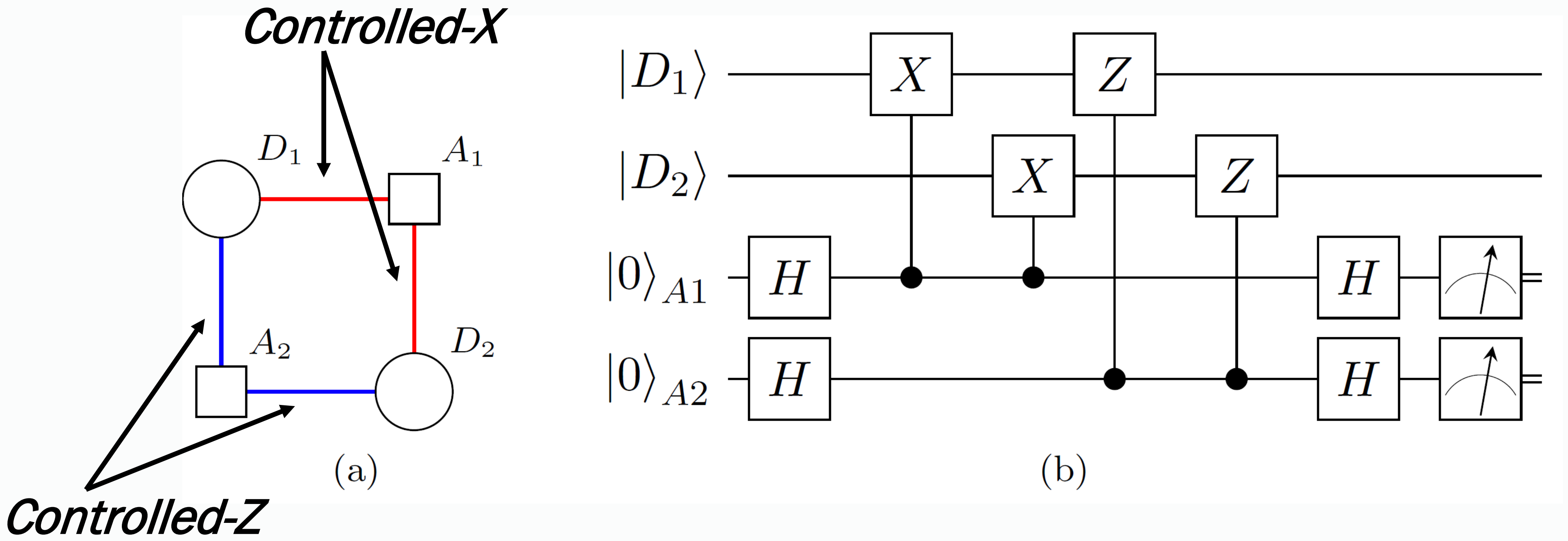
A. Fowler, et al., PhysRev A, 2012
J. Roffe, arXiv:1907.11157v1

Surface Code

- Based stabilizers and repeated measurements in both the Z and X bases
- Qubits classified as "data" or "measurement"
- Requirements:
  - All qubits must allow single-qubit rotations and CNOT between nearest neighbors
  - For Hadamard, must be able to SWAP state with neighbors
  - Measurement in the Z basis



Basic Four-Cycle



A_1 measures X_1X_2 (phase)

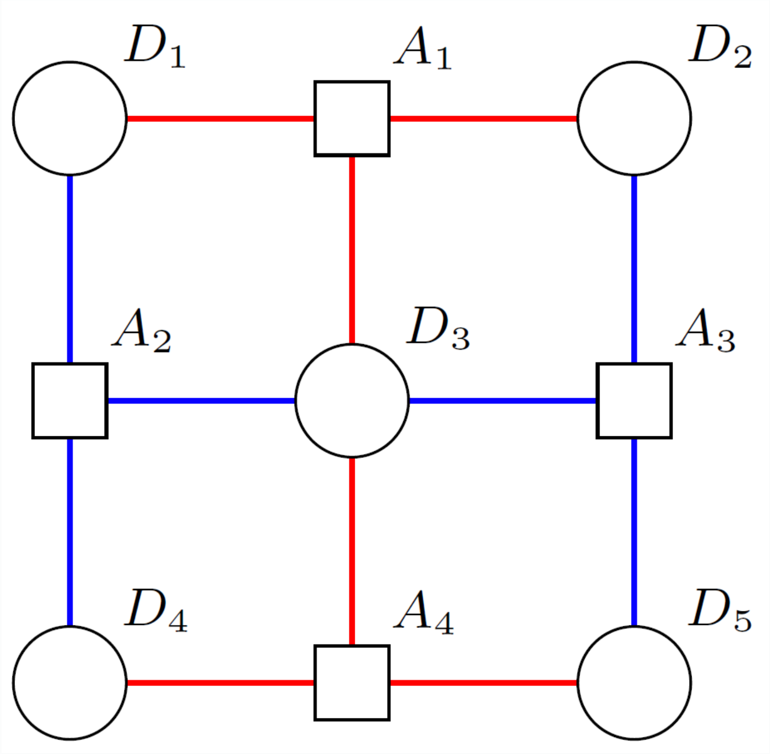
A_2 measures Z_1Z_2 (bit flip)

2 physical qubits (D), 2 stabilizers

n = 2, m = n - k = 2, which means **k = 0**

Not a useful code!

[[5,1,2]] Surface Code



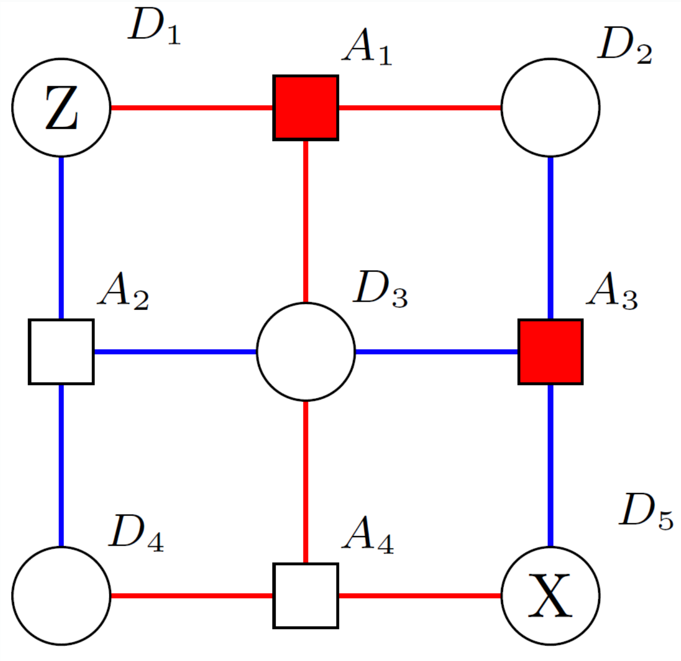
Stabilizers:

$A_1 = X_1X_2X_3$

$A_2 = Z_1Z_3Z_4$

$A_3 = Z_2Z_3Z_5$

$A_4 = X_3X_4X_5$

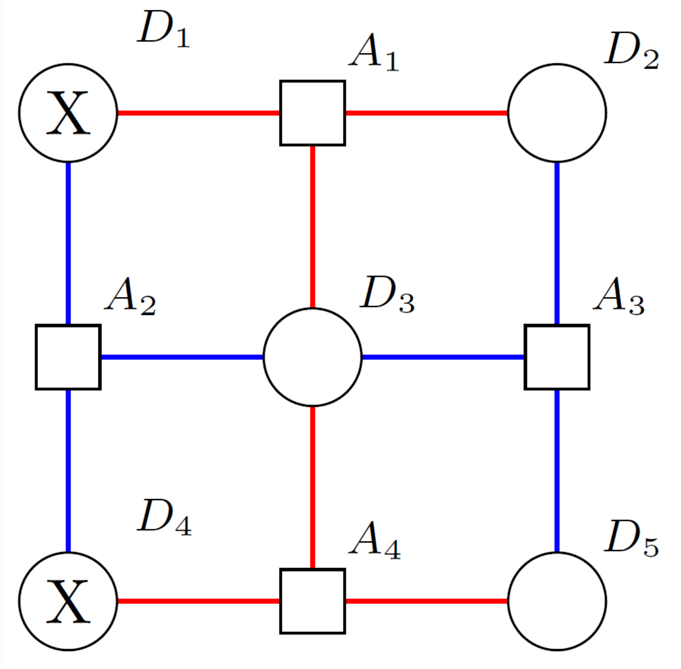


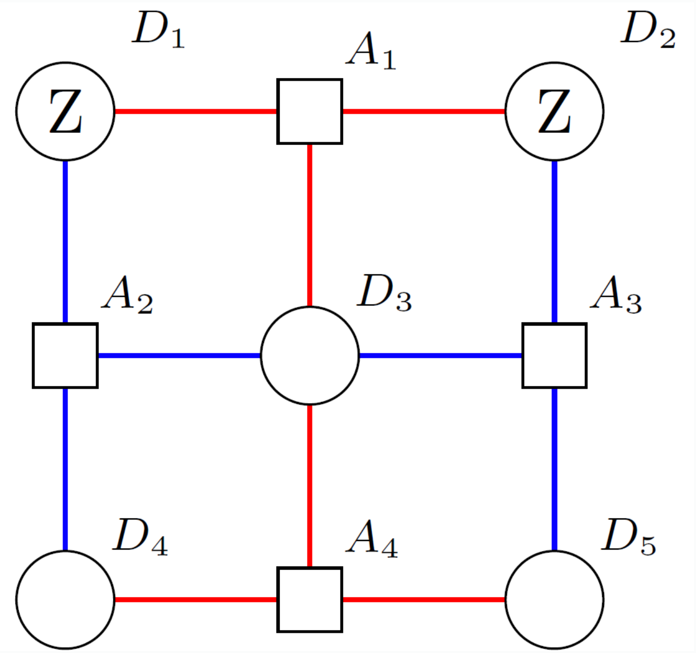
Detecting Errors

[[5,1,2]] Logical Operators

Logical X

Perform X along vertical border

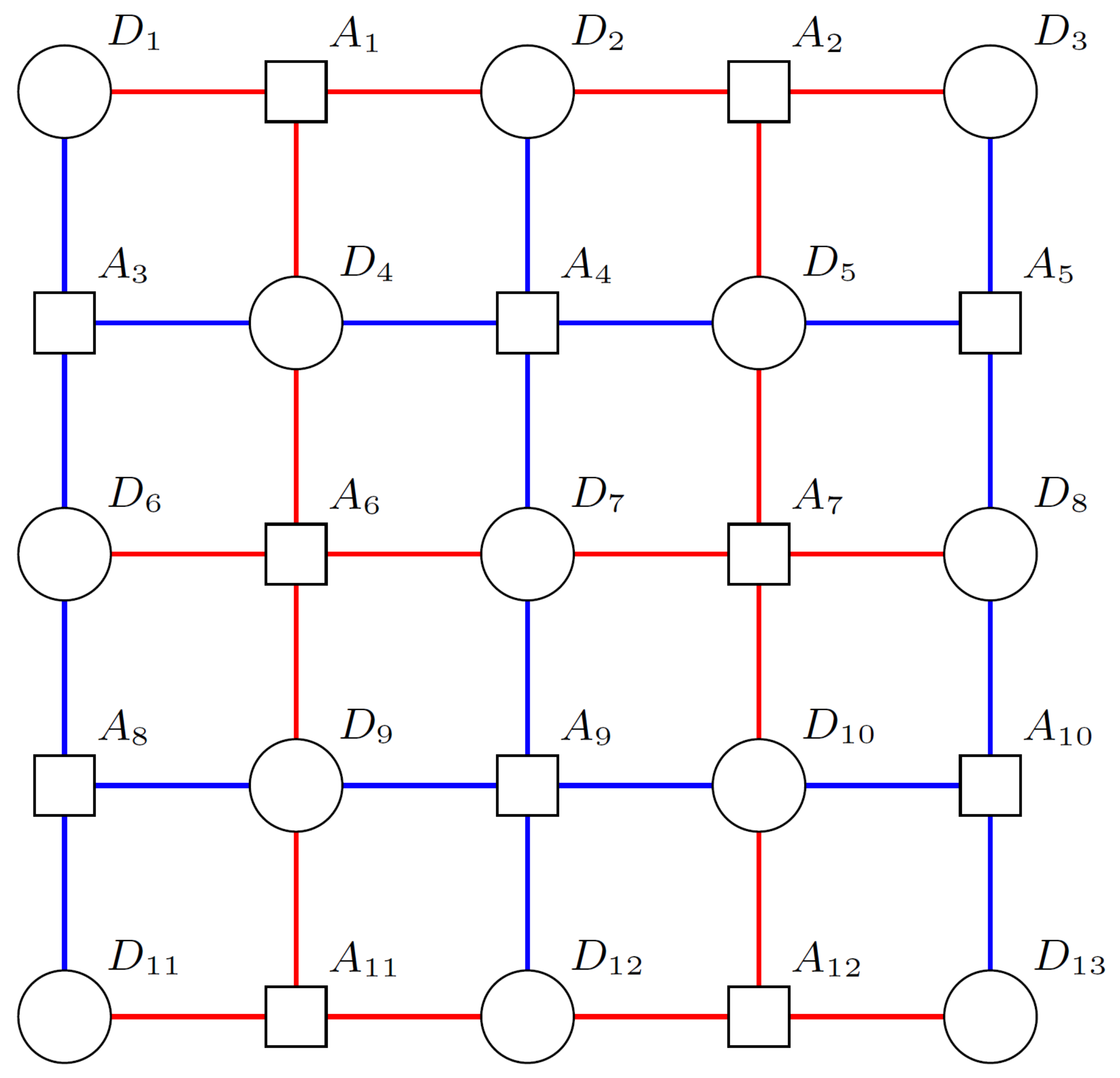




Logical Z

Perform Z along horizontal border

[[13,1,3]] Error Correcting Code



$$[[n = \lambda^2 + (\lambda - 1)^2, k = 1, d = \lambda]]$$

one logical qubit

Logical X

Perform X along vertical border

Logical Z

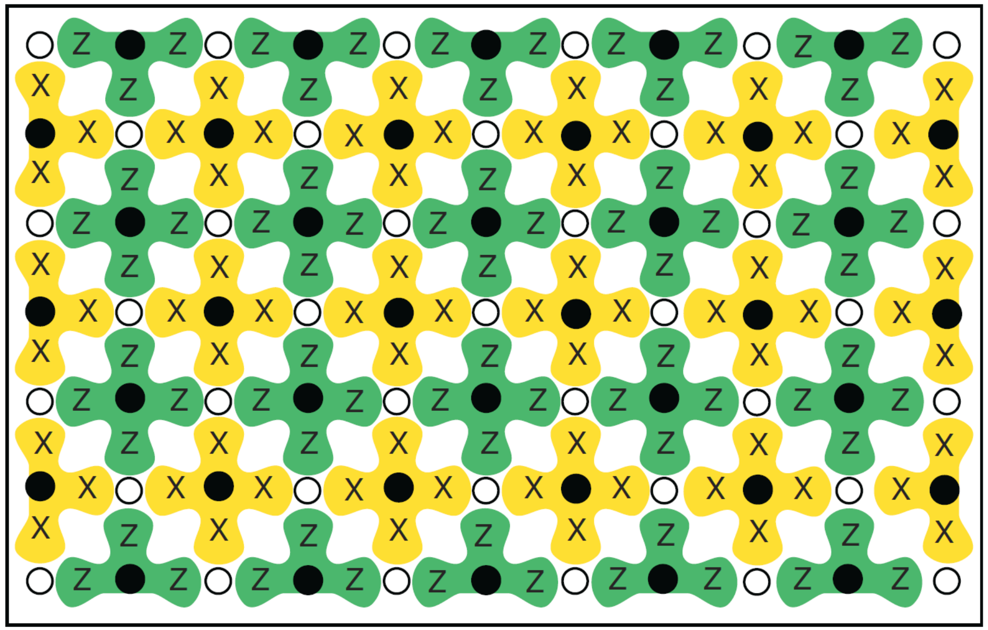
Perform Z along horizontal border

Measurement Qubits

- "measure-Z" qubits (green)
  "measure-X" qubits (yellow)

- measure-Z qubit forces its neighbors (a, b, c, d) into an eigenvalue of $Z_aZ_bZ_cZ_d$

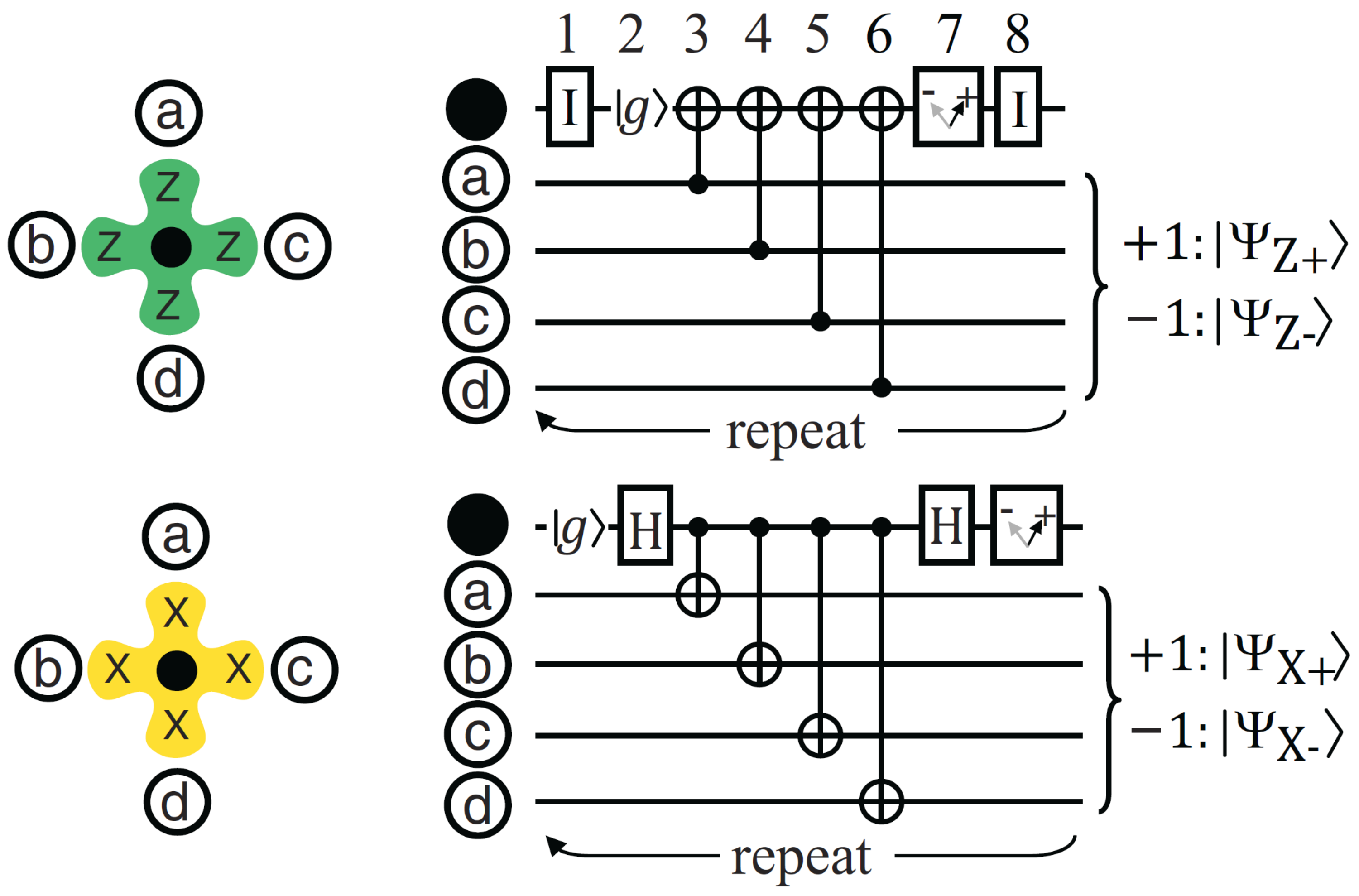
- measure-X qubit forces its neighbors (a, b, c, d) into an eigenvalue of $X_aX_bX_cX_d$



Stabilizers

Eigenvalue	$\hat{Z}_a\hat{Z}_b\hat{Z}_c\hat{Z}_d$	$\hat{X}_a\hat{X}_b\hat{X}_c\hat{X}_d$
$+1$	$\|gggg\rangle$	$\|++++\rangle$
	$\|ggee\rangle$	$\|++--\rangle$
	$\|geeg\rangle$	$\|+--+\rangle$
	$\|eegg\rangle$	$\|--++\rangle$
	$\|egge\rangle$	$\|-++-\rangle$
	$\|gege\rangle$	$\|+-+-\rangle$
	$\|egeg\rangle$	$\|-+-+\rangle$
	$\|eeee\rangle$	$\|----\rangle$
-1	$\|ggge\rangle$	$\|+++-\rangle$
	$\|ggeg\rangle$	$\|++-+\rangle$
	$\|gegg\rangle$	$\|+-++\rangle$
	$\|eggg\rangle$	$\|-+++\rangle$
	$\|geee\rangle$	$\|+---\rangle$
	$\|egee\rangle$	$\|-+--\rangle$
	$\|eege\rangle$	$\|--+-\rangle$
	$\|eeeg\rangle$	$\|---+\rangle$

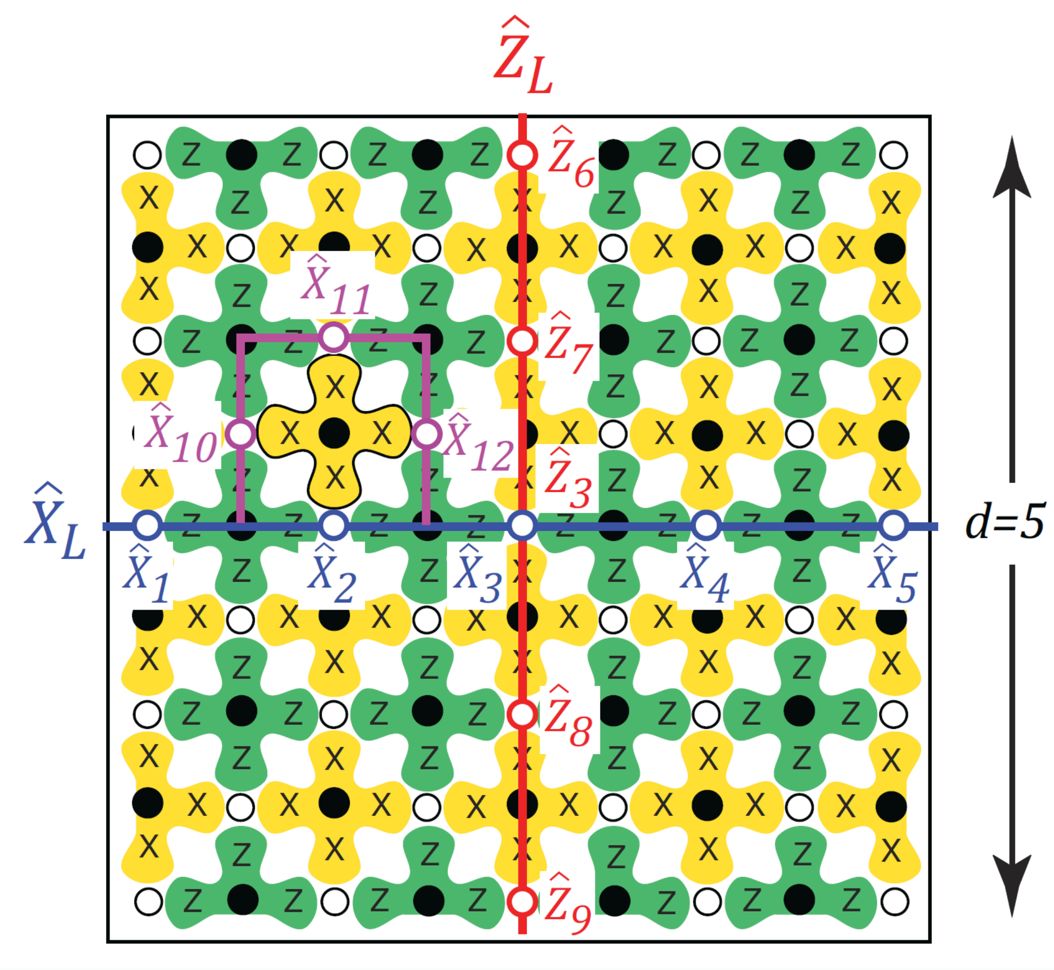
Surface Code Cycle



Error Detection

- Error causes changes in the measurement outcomes
- Does not try to correct (e.g. by applying X or Z)
  - Those operations are error-prone
- In software, tracks the errors in a qubit and adjusts subsequent measurement results (classically)
  - Later errors can "undo" the adjustment

Logical Operations



Missing measurement qubits (e.g., on boundaries) introduce additional degrees of freedom. Ex: Diagram has 41 data qubits and only 40 stabilizers.

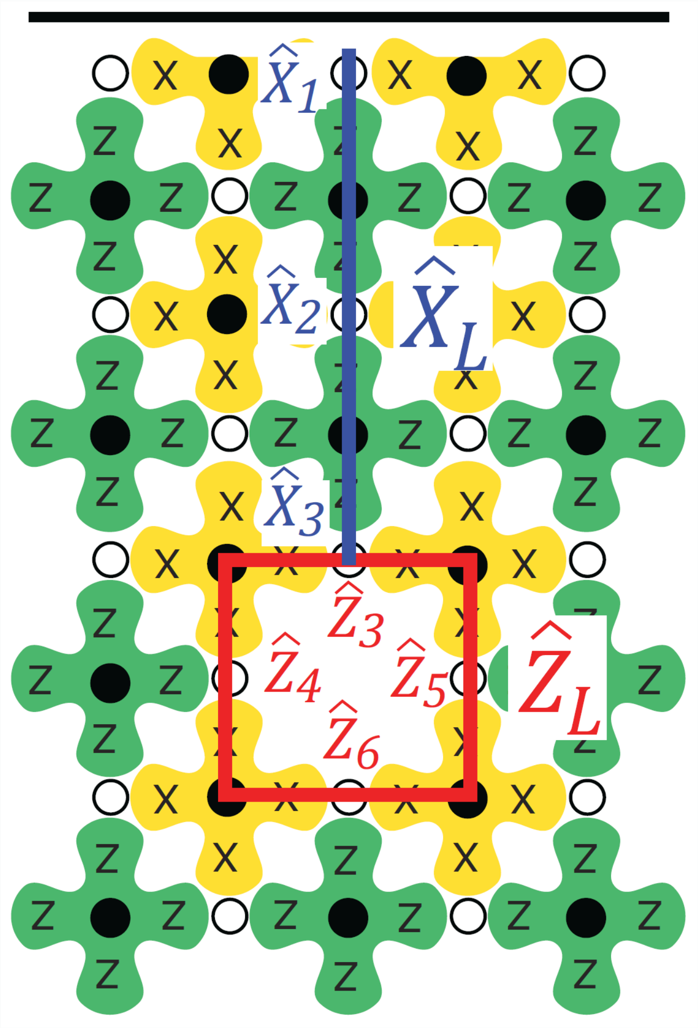
Can this array be viewed as a single logical qubit?

Applying X at ALL of the blue locations will alter the state of the array, but measurement results will remain the same. It has the affect applying a logical X to the logical qubit.

Likewise for Z operator applied along red line.

Large arrays are desired for low logical error rates. (More on this later.) How can we increase the number of logical qubits within an array?

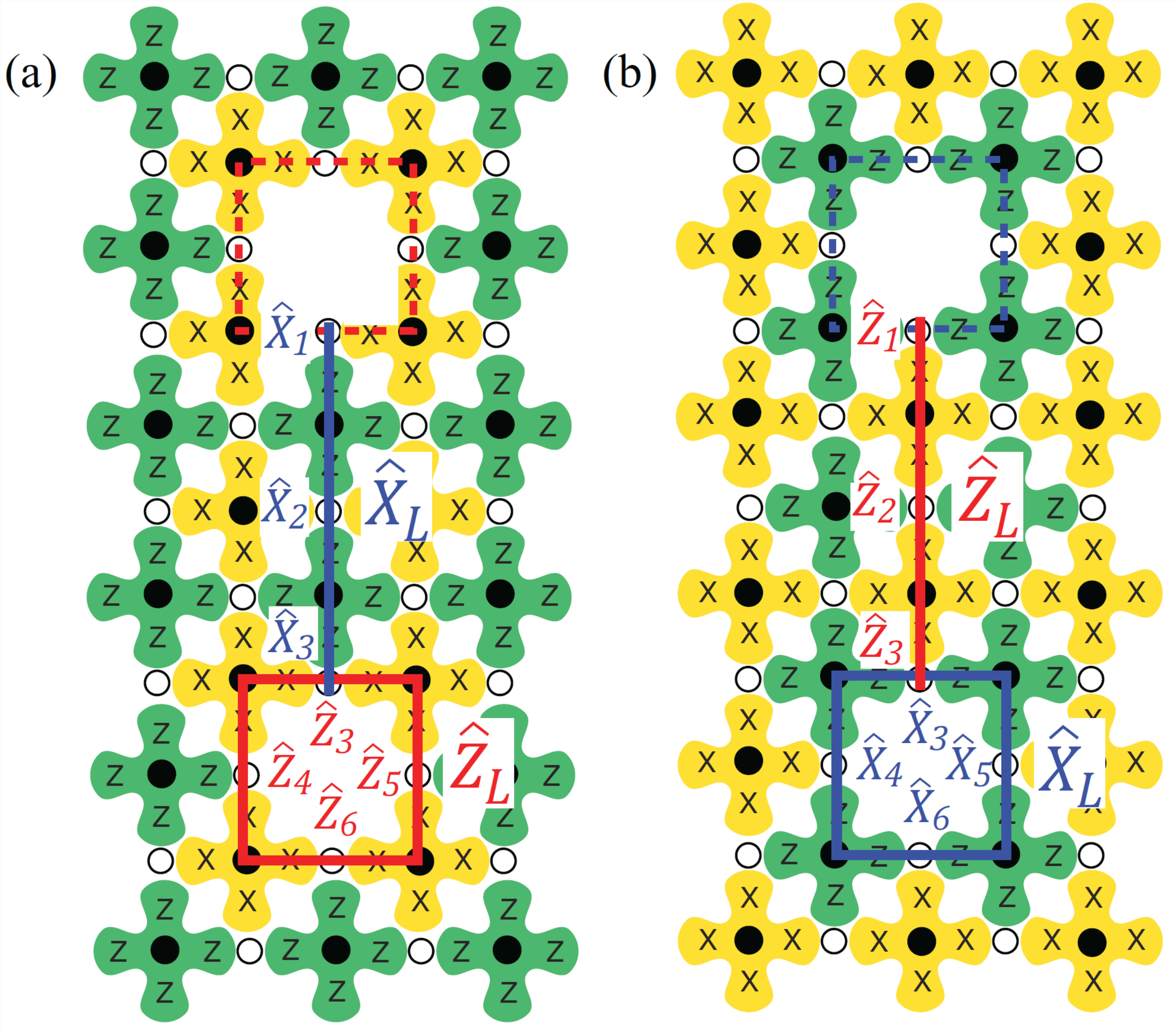
Logical Operations



Create "holes" (defects) to generate additional degrees of freedom. Just "turn off" one or more measurement bits.

Figure shows a "single Z-cut" qubit.

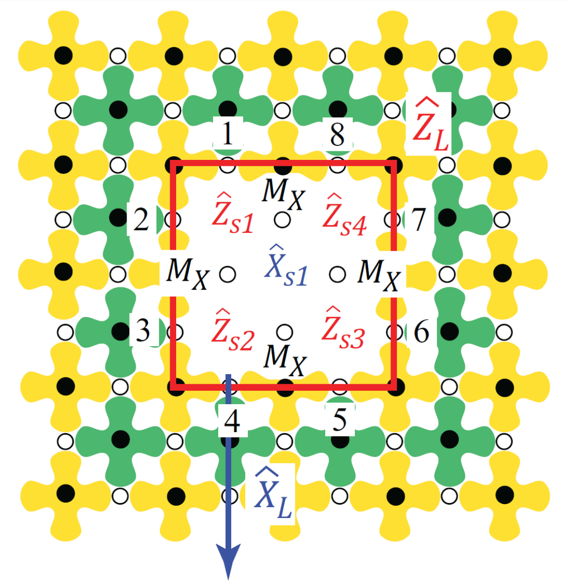
Logical Operators



These cuts result in d = 3.

Moving further apart increases the distance for X in figure (a), but Z is limited by the size of the cut.

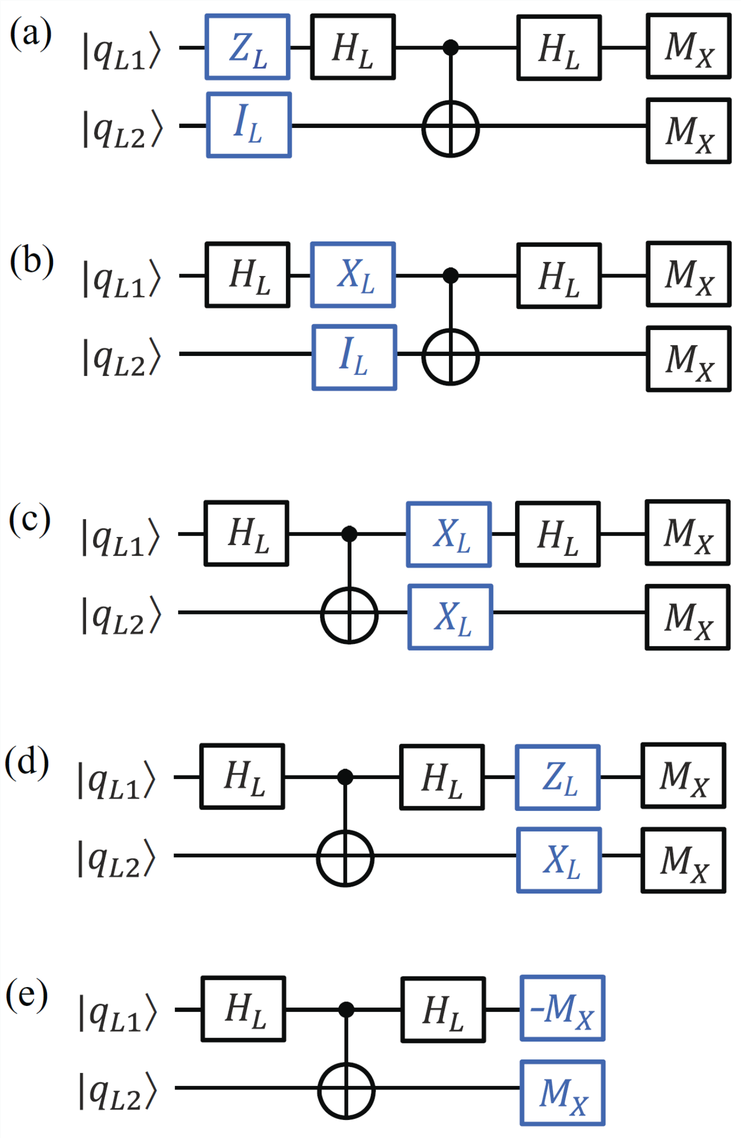
Can create larger holes and space further apart to increase d. (E.g., d=8, d=16)



Software "Gates"

Logical X and Z operators are implement in **software**

Commute through gates until reach another gate of the same type (cancel each other) or a measurement (correct the outcome)



Moving Qubits Around the 2D Array

- Can move the location of the holes around in the 2D surface code array
- This is helpful for implementing a logical CNOT gate between two logical qubits, by a braiding transformation

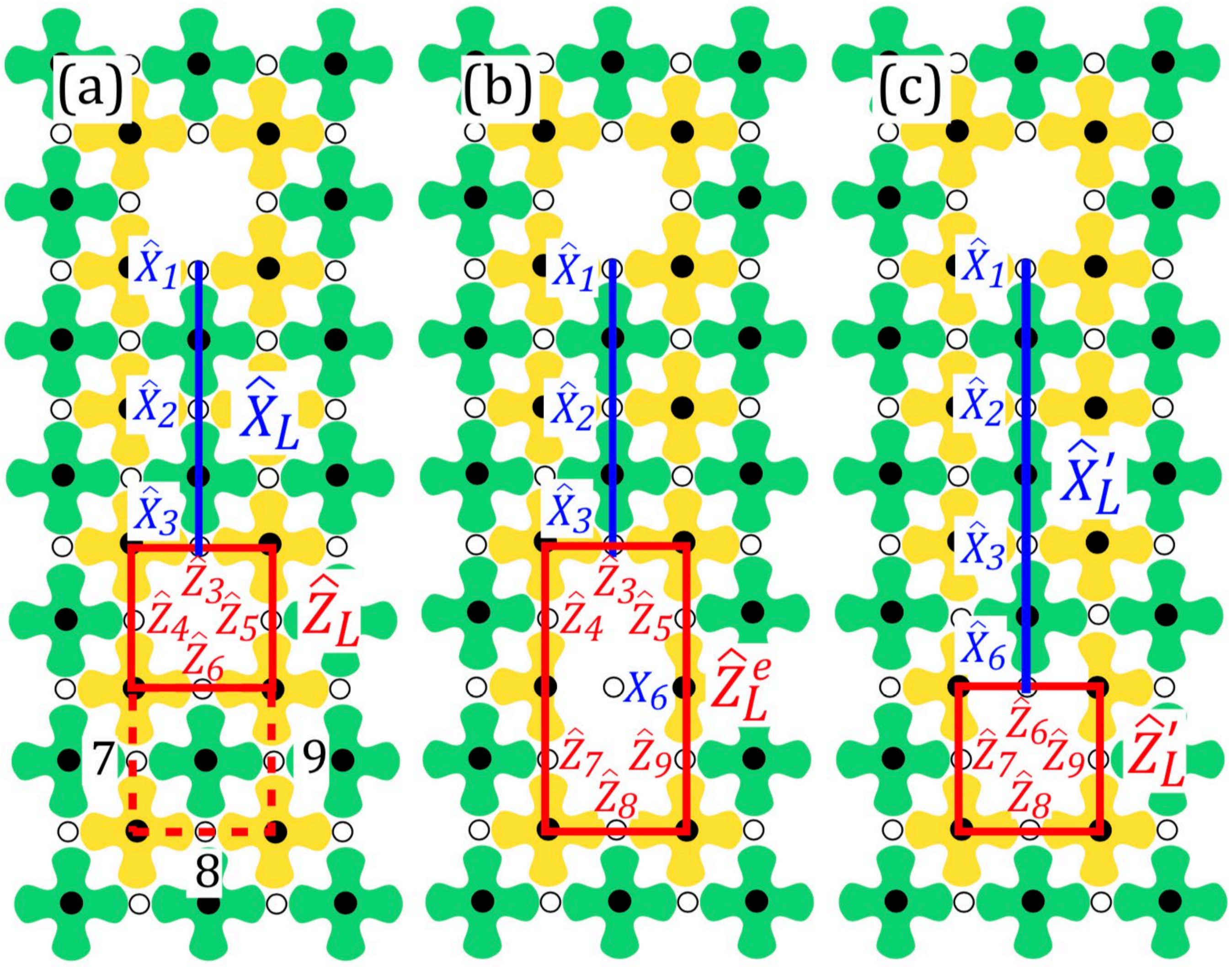
Next few slides borrowed from tutorial by Mark Wilde, LSU
https://www.scribd.com/presentation/413852728/Tutorial-on-Surface-Code-Quantum-Error-Correction

Moving Qubits Around the 2D Array: Example

a) Wait for current surface code cycle to complete.

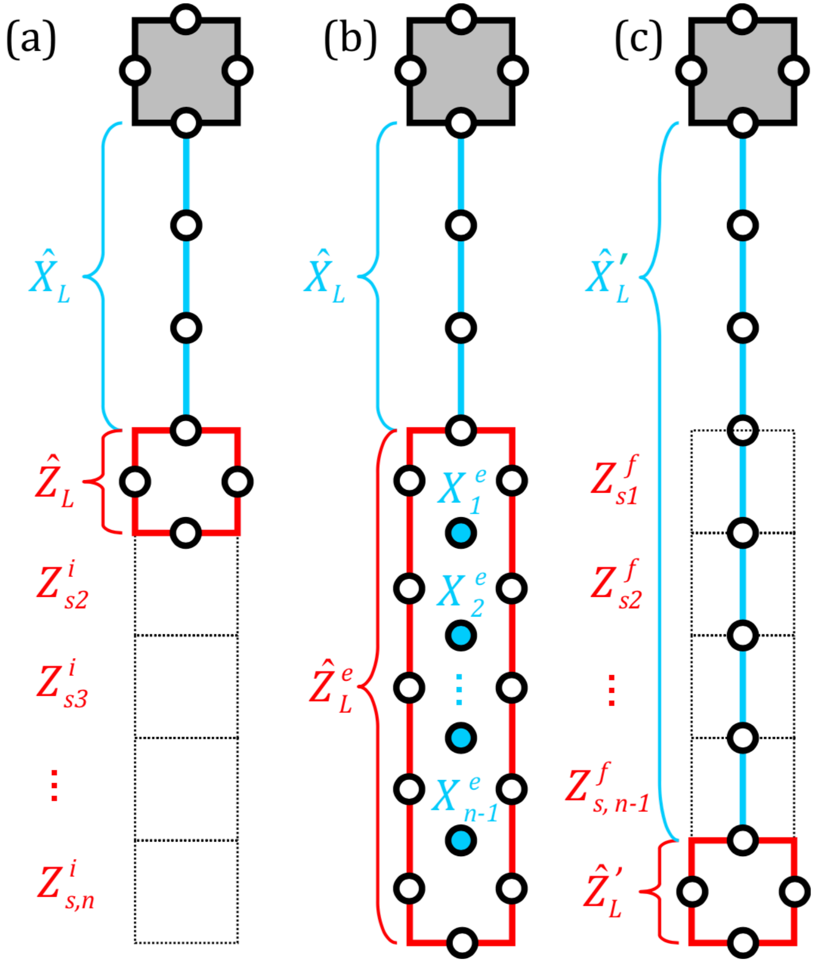
b) Then don't measure *Z*-stabilizer below *Z*-cut, and turn 4-terminal *X*-stabilizers acting on qubit 6 to 3-terminal stabilizers. Measure *X* of qubit 6

c) Turn on measure-*Z* qubit of original *Z*-cut hole. Convert *X* stabilizers back to 4 terminals



Multicell Move Extending One-Cell Move

- Multicell move is performed similarly to one cell move, and can be completed in a single surface code cycle



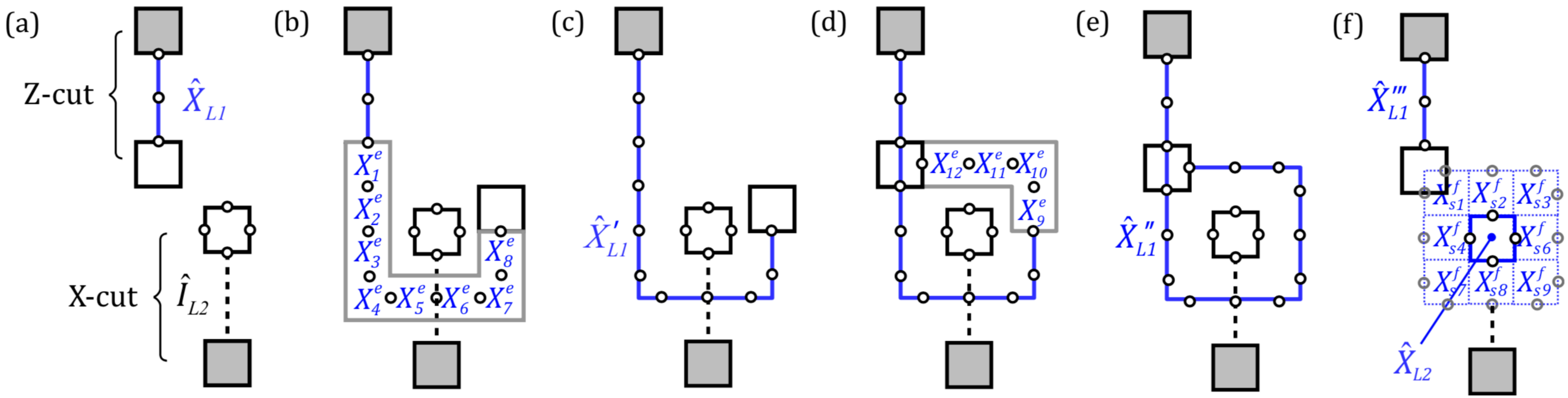
CNOT by Braiding (Sketch)

- Recall that CNOT in Heisenberg picture is equivalent to transformation of Pauli operators as

$$X \otimes I \to X \otimes X$$
$$Z \otimes I \to Z \otimes I$$
$$I \otimes X \to I \otimes X$$
$$I \otimes Z \to Z \otimes Z$$

- So we need to effect this transformation on logical operators

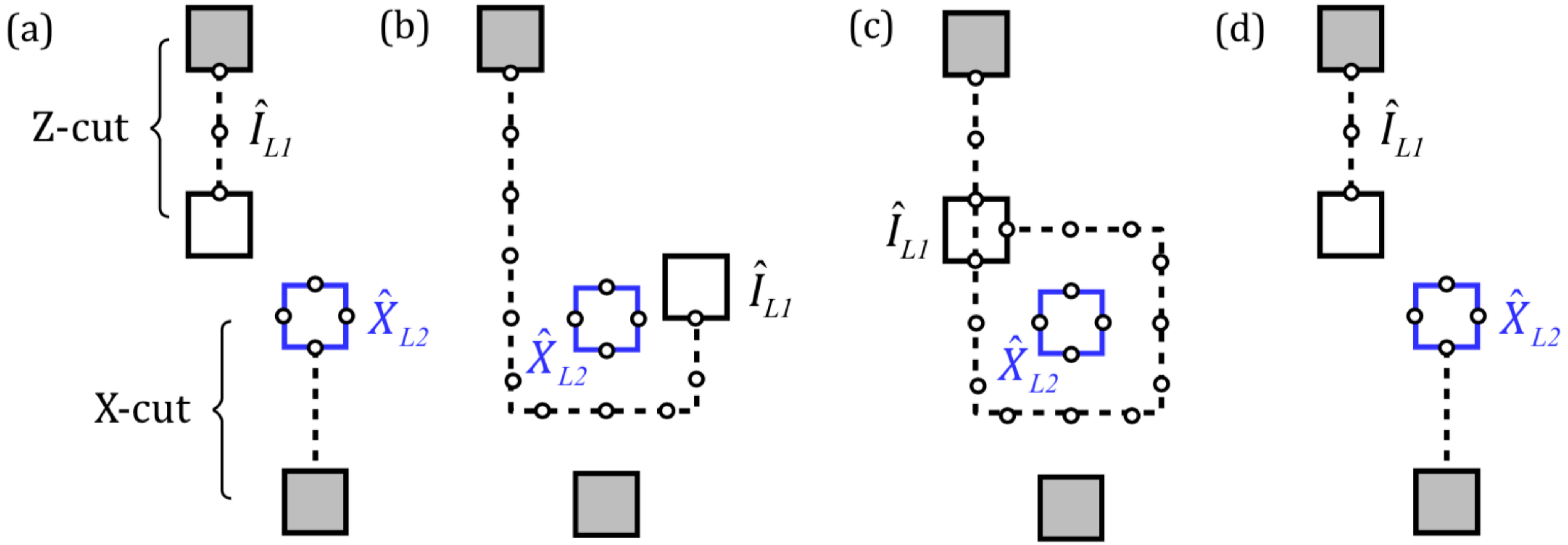
CNOT by Braiding (Sketch)



Effect of loop move around the X-cut of the upper hole of the second qubit is equivalent to realizing a logical X for the second qubit:

$$X \otimes I \rightarrow X \otimes X$$

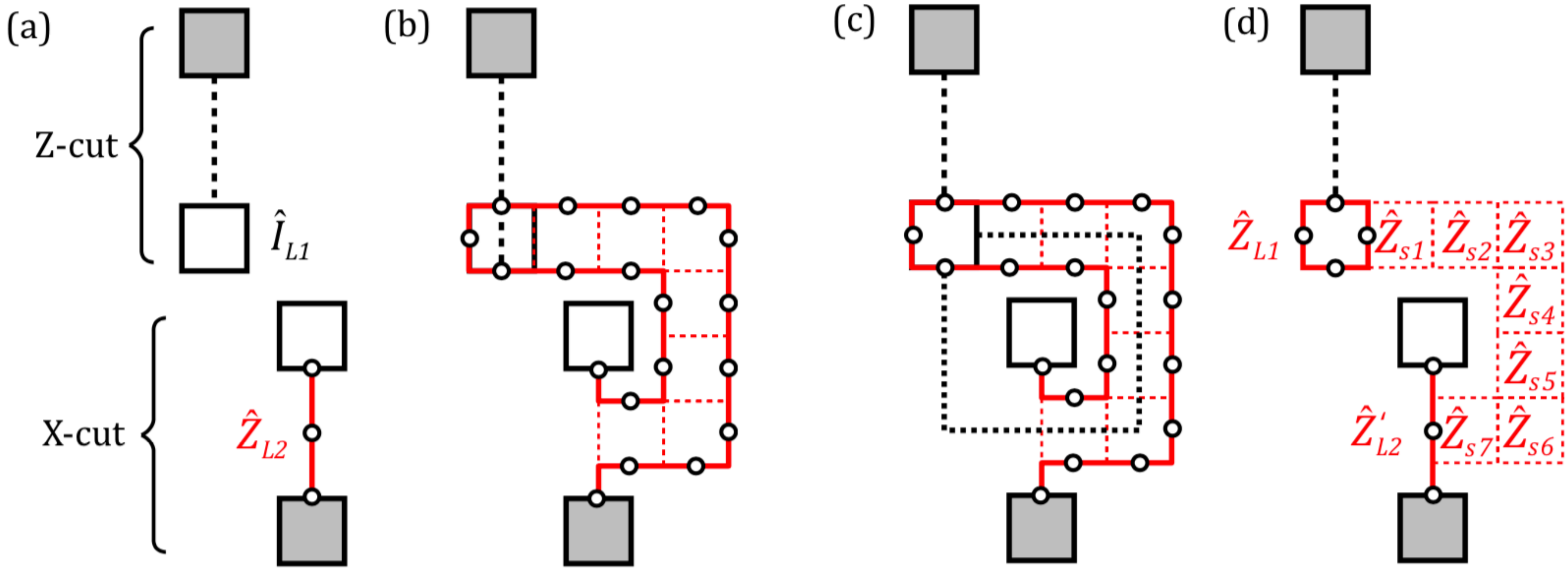
CNOT by Braiding (Sketch)



Effect of loop move for *I* on 1st qubit and *X* on 2nd has no effect on 2nd qubit:

$I \otimes X \rightarrow I \otimes X$

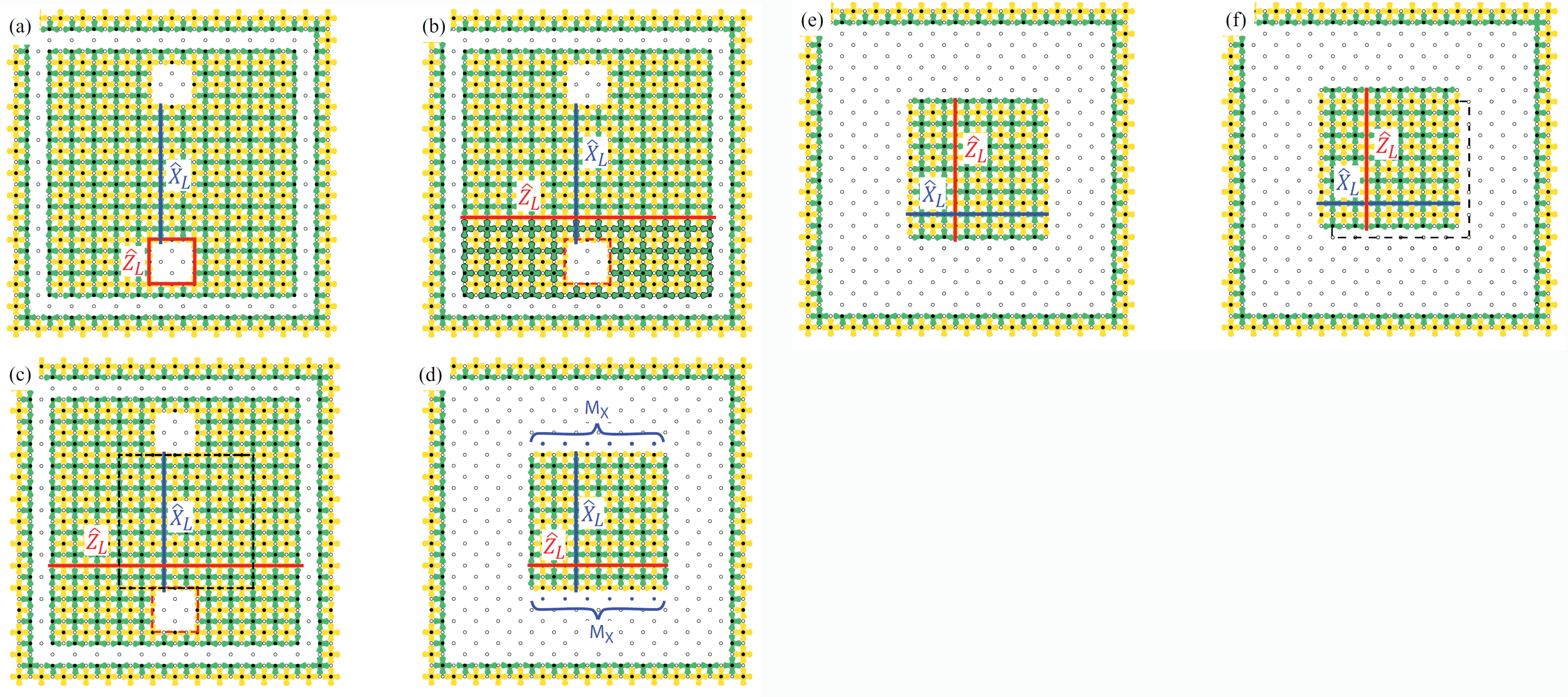
CNOT by Braiding (Sketch)



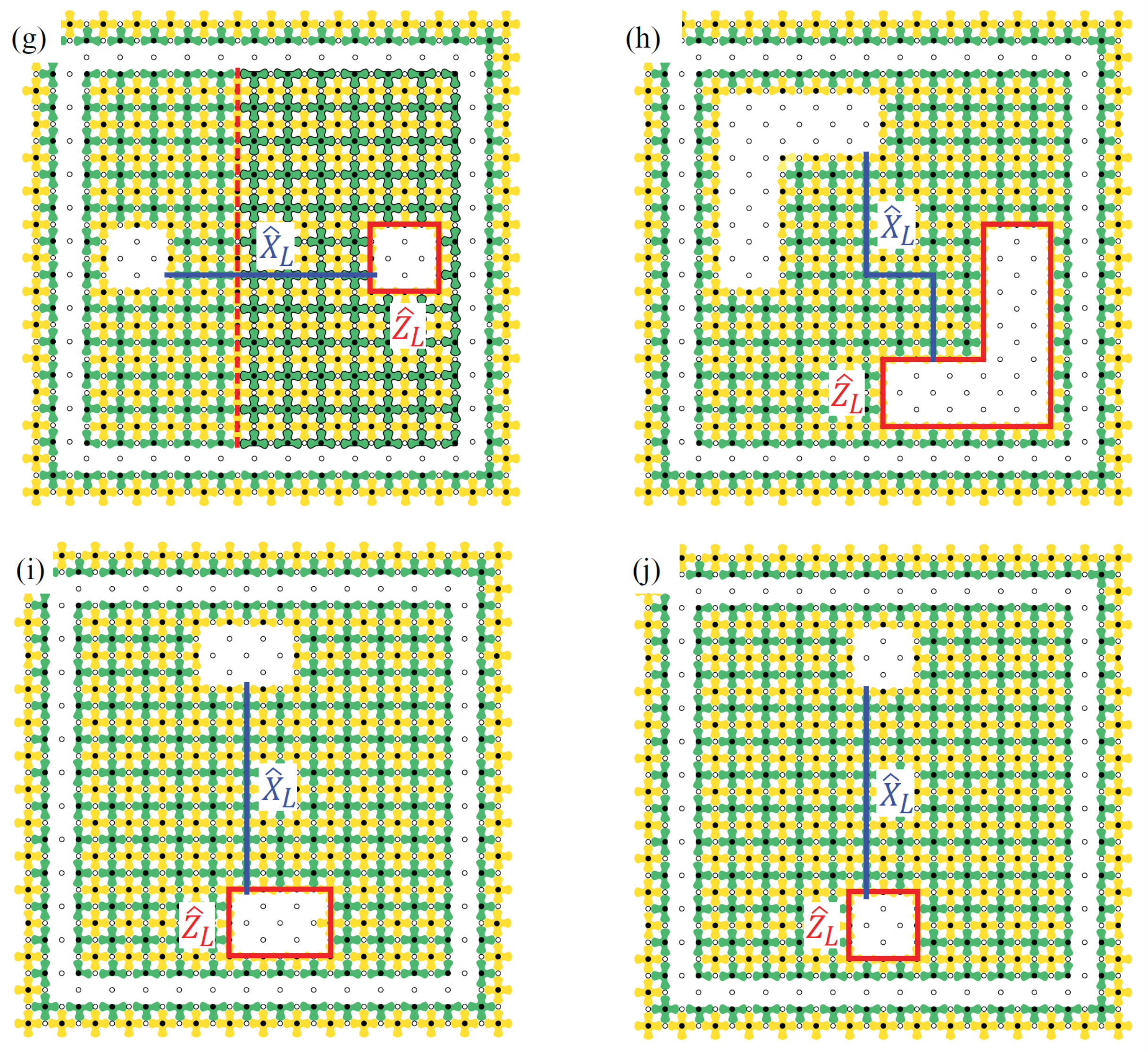
- Effect of loop move for *I* on 1st qubit and *Z* on 2nd affects 1st qubit:

$$I \otimes Z \rightarrow Z \otimes Z$$

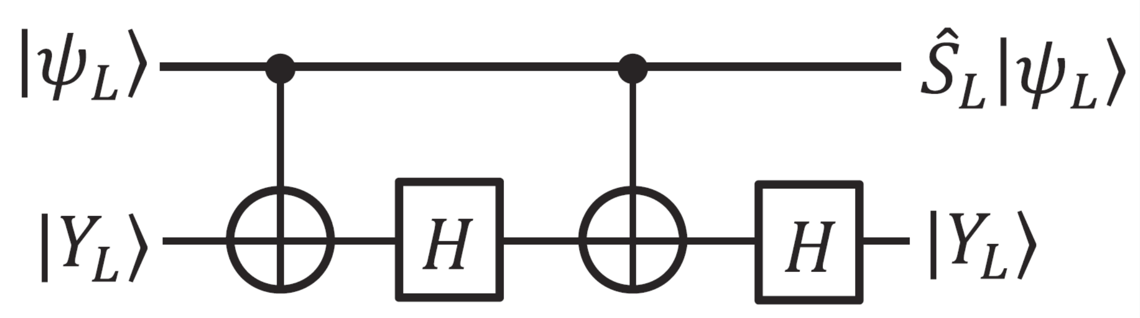
Hadamard (no details)



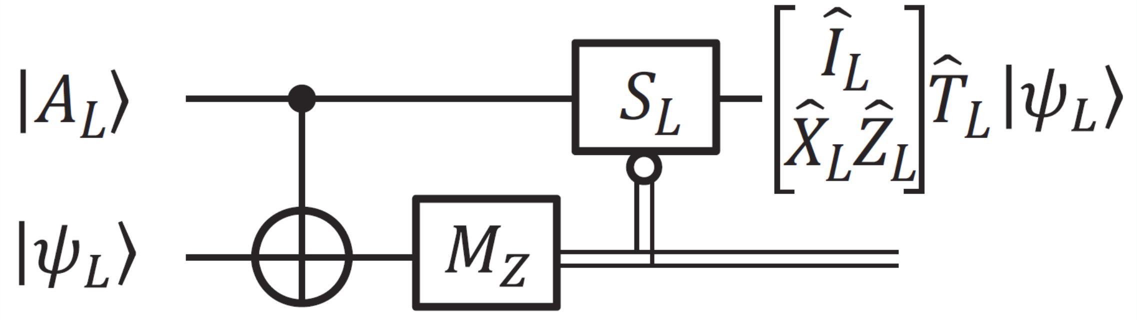
Hadamard (no details)



S and T Gates



$|\psi_L\rangle$
$\hat{S}_L|\psi_L\rangle$
$|Y_L\rangle$
H
H
$|Y_L\rangle$



$|A_L\rangle$
S_L
$\begin{bmatrix} \hat{I}_L \\ \hat{X}_L\hat{Z}_L \end{bmatrix}\hat{T}_L|\psi_L\rangle$
$|\psi_L\rangle$
M_Z

How many physical qubits?

Increasing d (dimension of array) improves logical error rate

On the order of **thousands** of physical qubits per logical qubit